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Algorithmic Graph Theory WS 02/03
www.inf.uni-konstanz.de/algo/lehre/ws02/gt

# Exercise Sheet 8

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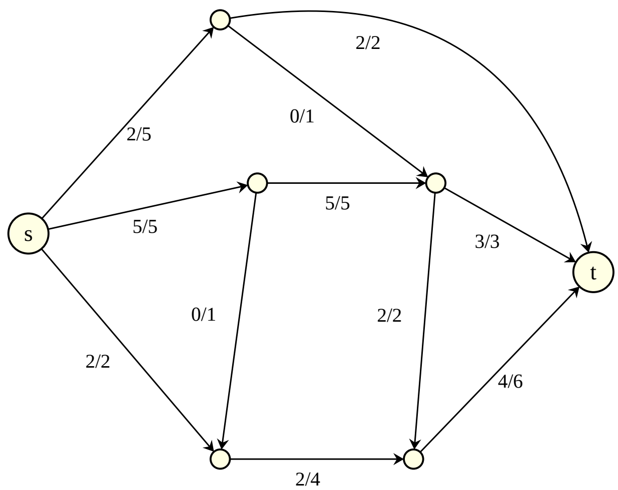
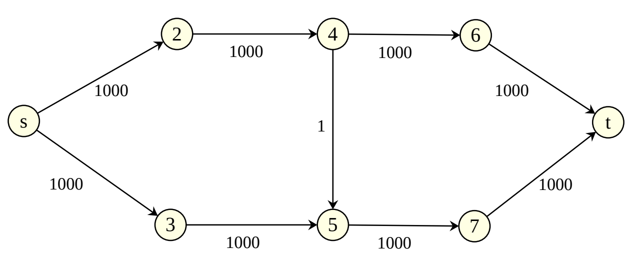
**Exercise class:** 13 February 2003

**Exercise 8.1:** Use the Ford-Fulkerson algorithm to find a maximum $s$-$t$-flow in the following flow network, starting with the initial flow (in the given flow network, $3/5$ means initial flow $3$ and capacity $5$.) Find also a minimum $s$-$t$-disconnecting set of edges that proves the maximality of the flow.




**Exercise 8.2** The Ford-Fulkerson algorithm is not deterministic since in the breadth-first search the node $u$ that is taken from the set $S$ in each step is taken arbitrarily from all unmarked nodes $v \in S$.

Applying the Ford-Fulkerson algorithm to the following flow network with initial flow equal to zero for every edge, what is in the worst case the number of augmenting paths that are determined during the algorithm (i.e., the maximum number of such paths)?



**Exercise 8.3:** Model the following problem as a graph colouring problem: Suppose you want to schedule final exams and, being very considerate, you want to avoid having a student do more than one exam a day. We shall call the courses 1,2,3,4,5,6,7. In the table below a star in entry ij means that course i and j have at least one student in common so you can't have them on the same day. What is the least number of days you need to schedule all the exams? Show how you would schedule the exams.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | | * | * | * | | * | * |
| 2 | * | | * | | | | * |
| 3 | * | * | | * | | | |
| 4 | * | | * | | * | * | |
| 5 | | | | * | | * | |
| 6 | * | | | * | * | | * |
| 7 | * | * | | | | * | |

**Exercise 8.4:** Show by induction that every simple and planar graph can be coloured with six colours.