Exercise Sheet 4

Issue date: 28 November 2002   **Hand in by** 10 December 2002
Exercise class: 12 December 2002

Exercise 4.1: Which of the following sequences are graphic? Provide a construction or a proof of impossibility for each.

   a) $(5, 5, 4, 3, 2, 2, 2, 1)$

   b) $(5, 5, 4, 4, 2, 2, 1, 1)$

   c) $(5, 5, 5, 3, 2, 2, 1, 1)$

   d) $(5, 5, 5, 4, 2, 1, 1, 1)$

Exercise 4.2 Suppose $d = (d_1, \ldots, d_{2k})$ is defined by $d_{2i} = d_{2i-1} = i$ for $1 \leq i \leq k$. Prove that d is graphic.
[Hint: Construct a graph with degree sequence d.]

Exercise 4.3: Let G be a Semi-Eulerian graph that is not Eulerian.

   a) Show that G has exactly *two* nodes s and t of odd degree, and the two endnodes of every Eulerian trail in G are s and t.

   b) How can G be modified to a graph G' only by duplicating edges of G such that the resulting graph G' is Eulerian?

   c) What is the minimum number of edges that have to be duplicated in b)?

Exercise 4.4: Every morning the lazy postman takes the bus to the post office. Starting there, he arranges his route so that he ends at home to go back to sleep as quickly as possible. Below is a map of the streets along which he must deliver mail, giving the number of minutes required to walk each street, whether delivering or not. P denotes the post office and H denotes home.
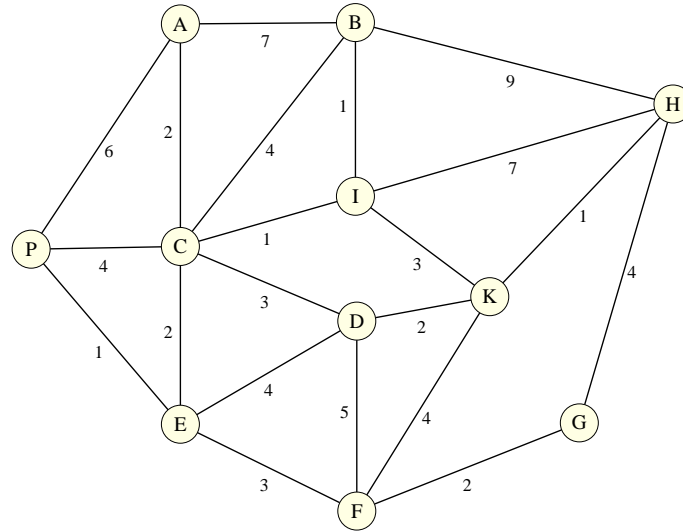
Which edges have to be traversed more than once in an optimal route (i.e., a shortest walk from P to H containing all edges)?