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Exercise Sheet 3

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Exercise 3.1: Let $G = (V, E)$ be a connected graph with real-valued edge weights $c(e)$ for $e \in E$. The following describes Borůvka's algorithm.

1. We maintain a set B of blue-coloured edges which is empty at the beginning.
2. Consider the subgraph G_B of G induced by the edges B (at the beginning it consists of $|V|$ isolated vertices.)
3. While G_B is not connected
   (a) Let $C_1, \dots C_k$ be the connected components of G_B. For each C_i choose one edge $e_i = \{u_i, v_i\}$ such that u_i belongs to C_i and v_i doesn't belong to C_i, and $c(e_i)$ is minimal among these edges. Note that for $i \neq j$ not necessarily $e_i \neq e_j$.
   (b) Colour all the edges chosen in (a) blue, i.e. add them to the set B.

Questions:

a) Find an example of a network containing 4 vertices and at least 4 edges such that the graph G_B computed by Borůvka's algorithm is a minimum spanning tree of G.

b) Prove the following statement: If a network is connected and the edge weights are pairwise different, then Borůvka's algorithm computes always a minimum spanning tree.

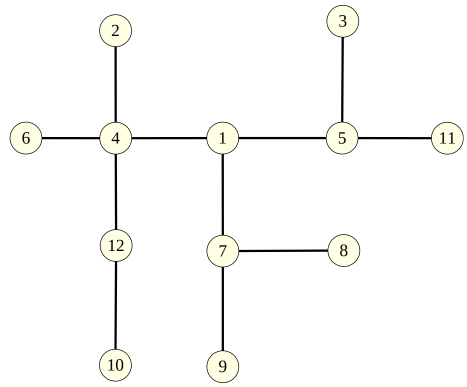
c) Is the condition in b) also necessary?

d) Modify the algorithm such that it computes always a minimum spanning tree, also for networks with possibly not pairwise different edge weights.

Exercise 3.2

a) Find the tree corresponding to the Prüfer sequence (2,5,5,11,8,8,2,5,11).

b) Find the Prüfer sequence of



Exercise 3.3: Calculate the number of spanning trees of the graph given by the following adjacency matrix:

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Exercise 3.4: Prove that from the matrix theorem it follows directly that the number of spanning trees on n vertices is n^{n-2}.