Exercise Sheet 2

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Exercise 2.1: Show that every tree is bipartite.

Exercise 2.2: For which $m, n \in \mathbb{N}_{>0}$ the complete bipartite graph $K_{m,n}$ is a tree?

Exercise 2.3: Let G be a simple connected graph with $d(v) = 2$ for all vertices v of G. Is G always a cycle?

Exercise 2.4: Let G be a simple graph. Prove the following equivalence:

$$G \text{ is a tree} \Leftrightarrow G \text{ contains no cycle and has exactly one spanning tree}$$

Exercise 2.5: Let G be a network, i.e. a graph where each edge e is associated with a real-valued cost c_e.

a) Give an example of a network that has two different minimum spanning trees.

b) Suppose now that the cost values of the edges are distinct, i.e. $e_1 \neq e_2 \Rightarrow c_{e_1} \neq c_{e_2}$. Show that then G has only one minimum spanning tree.