

# 26

## Social Networks

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Social networks provide a rich source of graph drawing problems, because they appear in an incredibly wide variety of forms and contexts. After sketching the scope of social network analysis, we establish some general principles for social network visualization before finally reviewing applications of, and challenges for, graph drawing methods in this area. Other accounts more generally relating to the status of visualization in social network analysis are given, e.g., in [Klo81, BKR<sup>+</sup>99, Fre00, Fre05, BKR06]. Surveys that are more comprehensive on information visualization approaches, interaction, and network applications from social media are given in [CM11, RF10, CY10].

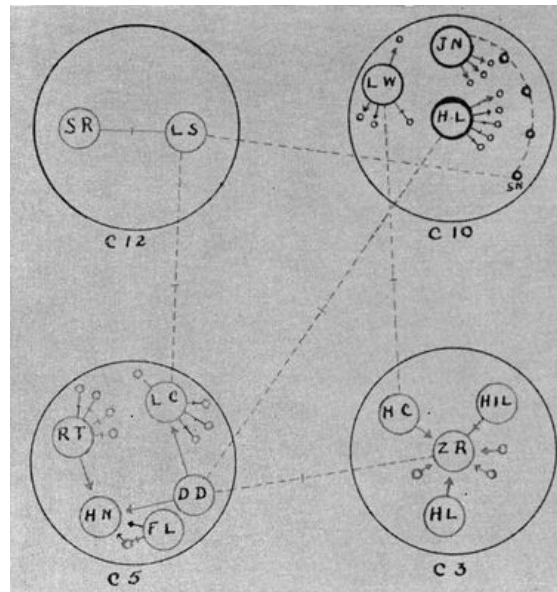
### 26.1 Social Network Analysis

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The fundamental assumption underlying social network theory is the idea that seemingly autonomous individuals and organizations are in fact embedded in social relations and interactions [BMBL09]. The term *social network* was coined to delineate the relational perspective from other research traditions on social groups and social categories [Bar54].

In general, a social network consists of *actors* (e.g., persons, organizations) and some form of (often, but not necessarily: social) *relation* among them. The network structure is usually modeled as a graph, in which vertices represent actors, and edges represent *ties*, i.e., the existence of a relation between two actors. Since traits of actors and ties may be important, both vertices and edges can have a multitude of attributes. We will use graph terminology for everything relating to the data model, and social network terminology when referring to substantive aspects.

While attributed graph models are indeed at the heart of formal treatments, it is worth noting that theoretically justified data models are not as obvious as it may seem [But09]. In fact, social network analysis is maturing into a paradigm of distinct structural theories and associated relational methods. General introductions and methodological overviews can be found in [WB88, WF94, Sco00, CSW05, BE05], a historic account in [Fre04a], and a comprehensive collection of influential articles in [Fre08].



**Figure 26.1** A sociogram from [Mor53, p. 422] showing a graph with fourteen highlighted vertices and four clusters.

In social network research it is important to clarify whether the networks are considered dependent or explanatory variables. In the former case the interest is in why and how networks form the way they do, and in the latter case the interest is in why and how networks influence other outcomes. For convenience, we will refer to the former as network theory (studying *network formation*) and to the latter as network analysis (studying *network effects*). A major distinction from non-network approaches is that the unit of analysis is the *dyad*, i.e. a pair of actors (may they be linked or not) rather than a monad (a singleton actor).

The methodological toolbox can be organized into the following main compartments.

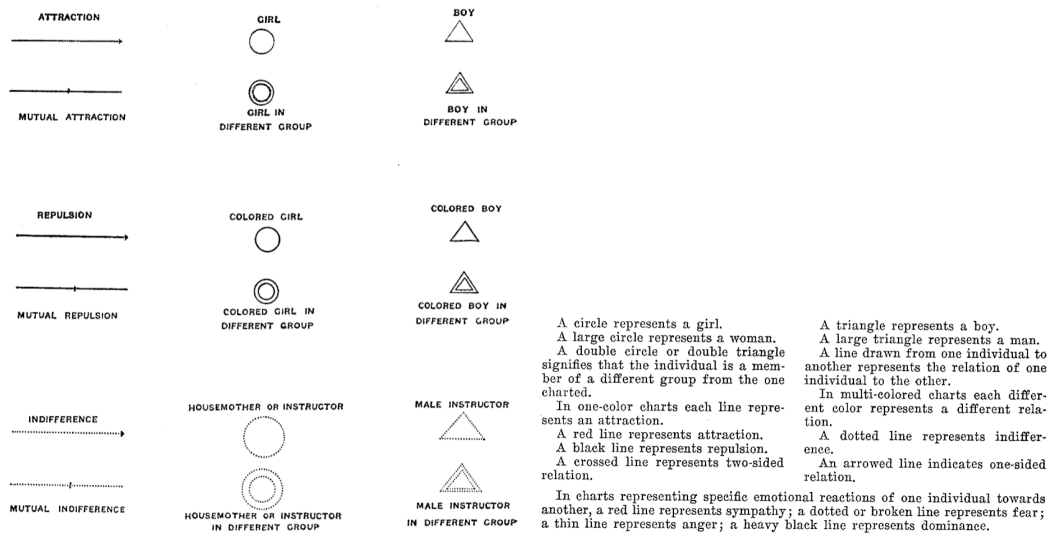
**Indexing** The assignment of values to predetermined substructures of any size. Most common are vertex, edge, and graph indices such as vertex centrality and graph centralization [Fre79], but sometimes the interest is also in evaluating larger substructures (e.g., group centrality) or the distribution of scores (e.g., degree distribution).

**Grouping** The identification of substructures and membership in them. Most common are decomposition into relatively dense subgraphs, partitions into equivalent positions [Ler05], and, more generally, blockmodeling [DBF05]. Other examples include subgraph counts (e.g., triad census) and various forms of domination and brokerage.

**Modeling** The use of statistical models for assessment and inference. Most common are modeling attempts to reproduce networks statistics, parameter estimation, and regression-type analyses.

Concrete examples of such methods are considered later in this chapter. Other important types of variation arise from special types of data such as longitudinal (temporal), multi-mode (multiple actor types), multiplex (multiple relation types), or multi-level (hierarchies of actors) data.

## KEY TO SOCIOGRAMS



**Figure 26.2** A notational system for sociograms [Mor53, p. 136].

Visualization has been instrumental in the study of social networks from the very beginning, and some historical examples are based on surprisingly sophisticated designs. The example in Figure 26.1 is from Moreno's book [Mor53], which is a rich source in this regard. In fact, he even specified a visual notation standard reproduced in Figure 26.2 (although neither graphical notation nor labeling are applied fully consistently), and introduced the terms *sociogram* (for a graphical representation of a social network) and *sociomatrix* (for a matrix representation of a social network).

## 26.2 Visualization Principles

Let us first establish a frame of reference for social network visualization based on a few organizing principles. We will then elaborate on various visualization approaches and the graph drawing problems they pose in Section 26.3.

The utility of a diagram is dependent on purpose and context. The two main purposes of network visualizations are *exploration* of data and *communication* of findings. The potential of using diagrammatic representations in the research process itself was stressed already by Moreno.

“A process of charting has been devised by the sociometrists, the sociogram, which is more than merely a method of presentation. It is first of all a method of exploration.” [Mor53, p. 95f]

A network diagram should therefore be designed to display the information relevant for an analytic perspective. As a consequence, there cannot be a single best way of representing social networks graphically, which in turn creates lots of opportunities for visualization and algorithm design. For concreteness, we give one striking example.

### 26.2.1 Illustrative Example

The following social network study [Kra96] has been used for the same purpose several times [BRW01, Bra08]. The study was conducted in an internal auditing unit of a large industrial company when organizational changes introduced by the newly assigned manager did not improve the unit's performance.

The formal hierarchy is shown in Figure 26.3(a), where made-up names are used to identify employees. To assess the internal functioning of the group, employees were asked who they would turn to for work related questions. The data obtained is an asymmetric advice network,

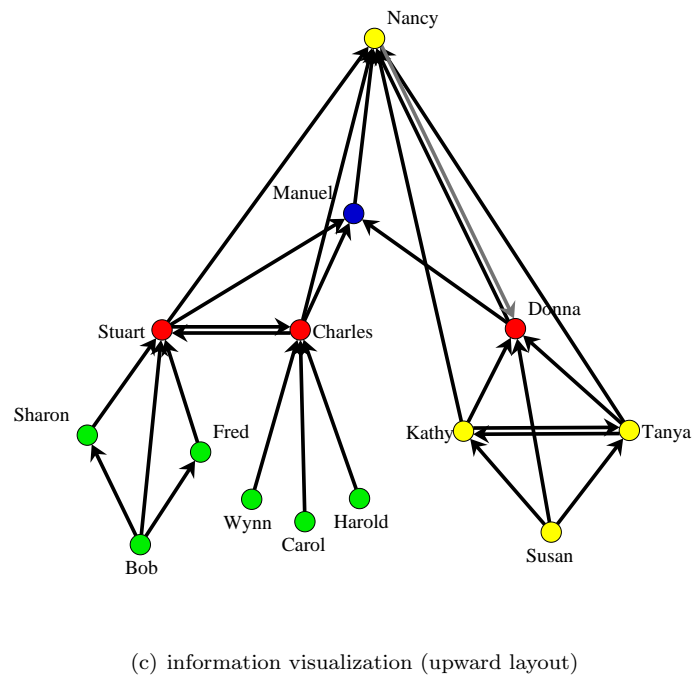
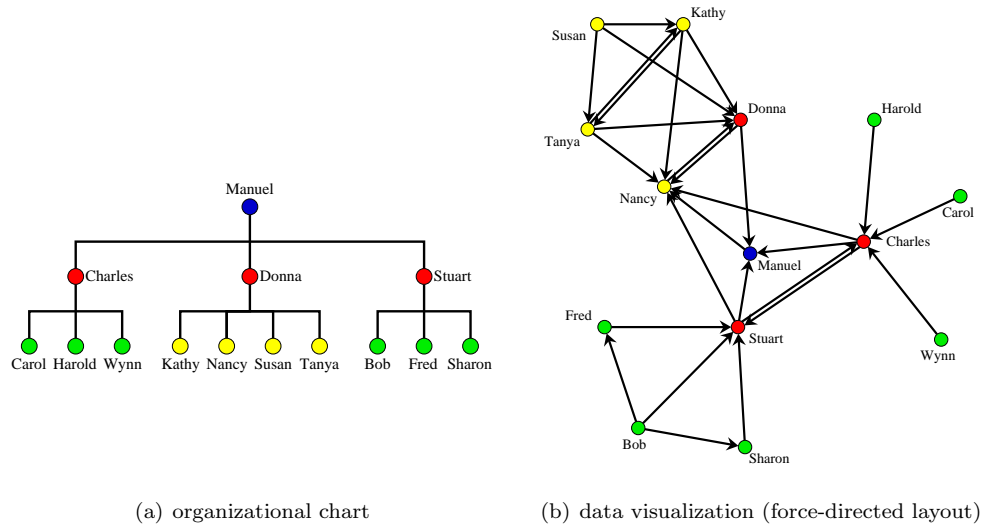
Manuel	.	0	0	0	0	0	0	0	0	0	0	1	0	0	<i>manager</i>
Charles	1	.	0	1	0	0	0	0	0	0	0	0	1	0	
Donna	1	0	.	0	0	0	0	0	0	0	0	0	1	0	<i>supervisors</i>
Stuart	1	1	0	.	0	0	0	0	0	0	0	0	1	0	
Bob	0	0	0	1	.	0	1	0	1	0	0	0	0	0	
Carol	0	1	0	0	0	.	0	0	0	0	0	0	0	0	
Fred	0	0	0	1	0	0	.	0	0	0	0	0	0	0	
Harold	0	1	0	0	0	0	0	.	0	0	0	0	0	0	<i>auditors</i>
Sharon	0	0	0	1	0	0	0	0	.	0	0	0	0	0	
Wynn	0	1	0	0	0	0	0	0	0	.	0	0	0	0	
Kathy	0	0	1	0	0	0	0	0	0	0	.	1	0	1	
Nancy	0	0	1	0	0	0	0	0	0	0	0	.	0	0	
Susan	0	0	1	0	0	0	0	0	0	0	1	0	.	1	<i>secretaries</i>
Tanya	0	0	1	0	0	0	0	0	0	0	1	1	0	.	

and the resulting directed network is shown in Figure 26.3(b). While the data is represented with clarity, there are no obvious implications. The situation becomes much more comprehensible when we understand that the rationale for looking at the advice network is the identification of an informal work hierarchy. Rearranging the vertices such that a maximum number of edges is directed upwards (and thus aligns with what can be assumed to be an informal work hierarchy) as in Figure 26.3(c) yields a strikingly clear picture with a single relation not in accordance with the informal hierarchy (but the formal): everyone, directly or indirectly, and including the manager himself, is seeking advice from Nancy, a secretary that was dismissive of the changes introduced. (Of course, after seeing a picture similar to this, the manager sat down with her, discussed her reservations and made sure that she understood his good intentions and the long-term benefits of his plans, thus turning the situation around.)

### 26.2.2 Substance, Design, Algorithm

The example above illustrates the importance of considering three key aspects in social network visualization [BKR<sup>+</sup>99].

**Substance** In general, the information to be conveyed in a network visualization is more than just the underlying graph. The substantive interest of those who collected network data typically necessitates the inclusion of attributes. Moreover, additional data may have been generated as the result of an analysis. In the above example, the substance of interest is the informal hierarchy within a business unit, and only by considering it in the design of the visualization, the diagram becomes informative. Through the appreciation of relevant substance, i.e. the application-specific contexts and interests, data visualizations are turned into information visualizations.



**Figure 26.3** Organizational chart and advice network in a business unit (adapted from [Kra96]).

**Design** Visualization design is the specification of a mapping from an information space to its graphical representation. The core choices are in assigning graphical elements such as points, lines, and areas to data objects, and in defining their graphical attributes such as position, shape, size, color, and so on such that the information is perceived correctly (effectiveness) and with low cognitive effort (efficiency).

Many overlapping and contradicting criteria need to be considered. In particular, Tufte advocates general information design criteria such as parsimony and accuracy [Tuf83, p. 51]. An important readability criterion is the avoidance of crossings [PCJ97], although finer distinctions may require more research: in a statement on the accurate representation of substance,

“The simplest, most efficient construction is one which presents the fewest meaningless intersections, while preserving the groupings, positions, or potential orders contained in the component...” [Ber83, p. 271],

Bertin acknowledges implicitly that crossings may also be meaningful. The efficiency of information visualizations, and network visualizations in particular, is a wide open field. A recent suggestion includes the assessment of cognitive load in user studies [HEH09].

**Algorithm** Suitable design is not necessarily realizable. Locally plausible design choices such as certain desired edge lengths may be interdependent and even contradicting. In the advice network example, the goal to direct as many edges upward as possible corresponds to an  $\mathcal{NP}$ -hard problem (FEEDBACK ARC SET) and may have multiple solutions of which a visualization will represent only one. Both, approximate and non-representative, solutions are highly problematic. If the advice network had been a directed cycle, all cyclic permutations represent equally good solutions to the upward drawing problem, because only one edge is pointing downward. Since each time a different actor ends up on top and only one such permutation is represented, the drawing provides a rather selective perspective. This is of course not due to the computational complexity of reversing the least number of edges; even if the advice network has an acyclic underlying graph, the vertical order is not defined uniquely. Note that in Figure 26.3(c), all secretaries could have been placed higher than the supervisors of the two auditing teams without reversing an edge.

Computational complexity as well as existence of solutions, their non-uniqueness, and the possibility of artifacts therefore place major restrictions on possible designs.

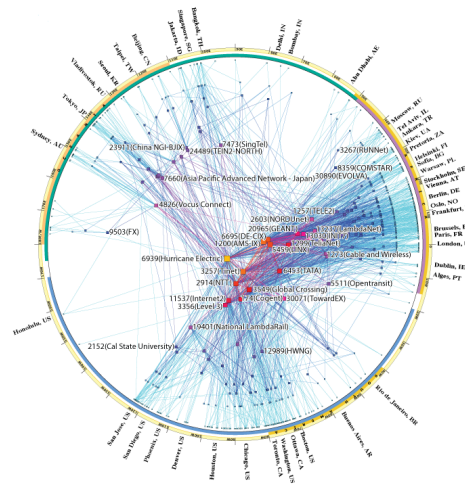
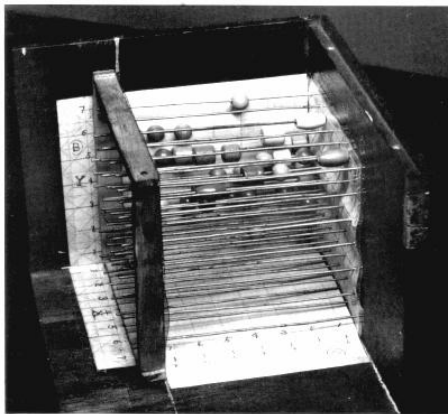
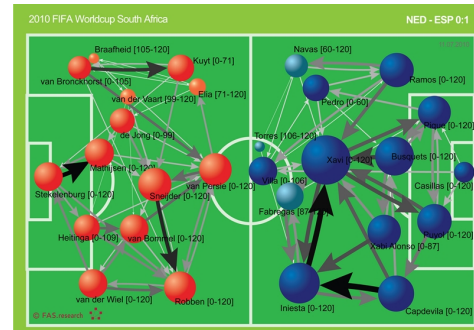
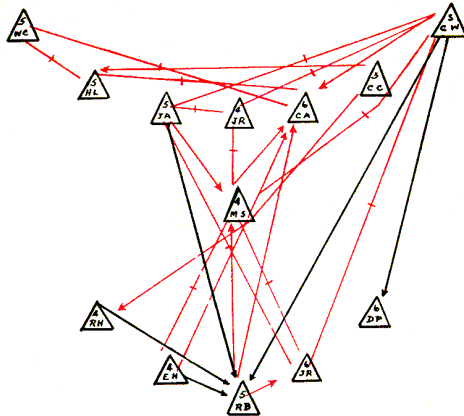
The richness of substantive interests and the need for substance-based designs thus creates immense potential for graph layout algorithms tailored to social networks.

## 26.3 Substance-based Designs

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Depending on data, substantive interest, and presentation context, very different designs are required for effective and efficient exploration and communication of social network information. Depending on the point of view, this is either a major burden or a horn of plenty for algorithmic and design challenges.

In this section, we review examples of substance-based designs and corresponding graph drawing approaches to demonstrate the richness of both, existing approaches and open



**Figure 26.4** Network visualizations in which coordinates are not defined by a graph drawing algorithm.

problems. Our selection of examples is, of course, heavily biased by their algorithmic interestingness. Figure 26.4 shows examples for designs we have excluded, because they do not require graph layout algorithms (although labeling and edge routing may be an issue).

We concentrate on two important analytical concepts (prominence and cohesion), and two data categories (two-mode and dynamic data). The designs we deal with are largely based on intuition and plausibility rather than perceptual and cognitive theories and empirical evidence. Among the few attempts to evaluate effectiveness and efficiency of network visualization design are [Win94, PCJ97, MBK97, HHE07, HEH09].

### 26.3.1 Prominence

Prominence indices  $p : V \rightarrow \mathbb{R}_{\geq 0}$  are used to rank the vertices  $V$  according to their structural importance [KB83]. Since there is no unanimity about their conceptual foundations [Fre79, Fri91], numerous such indices exist, and their properties vary tremendously. Although terminology is not well-defined and more refined classifications exist [Bor05, BE06], we distinguish only two groups based on geometric metaphors frequently invoked in their interpretation.

#### Status

It is commonplace to differentiate status into “high” and “low,” so that it seems almost mandatory to exploit this geometric interpretation for network visualizations. Not surprisingly, the apparent correspondence between substantive and geometric intuition has been used in the design of network diagrams already in times when no layout algorithms were available to social scientists. Figure 26.5 shows two historic examples, one with an extrinsic status attribute, and another with an intrinsic, structural one.

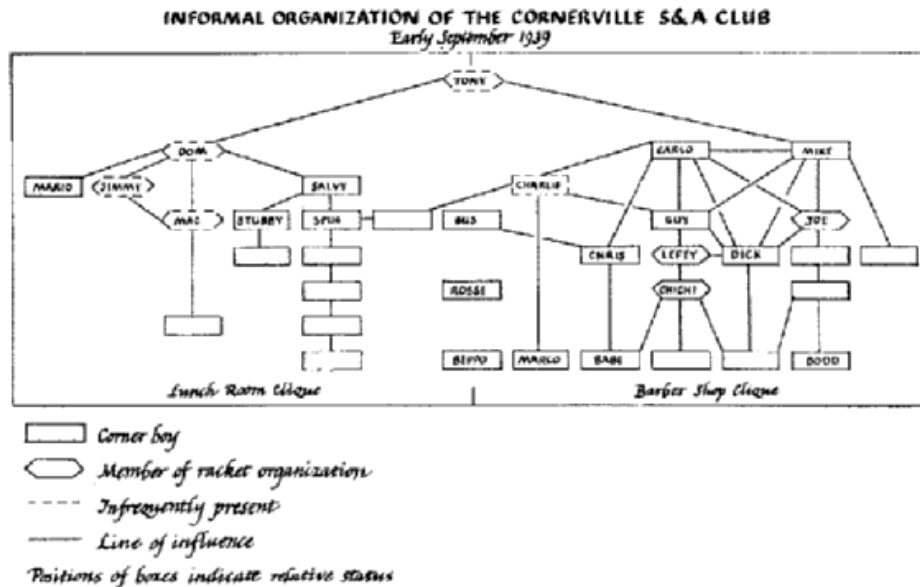
The advice network used for illustration above is an example in which a status hierarchy is conceived as emerging from an informal advice-seeking relation. Indeed, status is often analyzed in networks with directed edges, and because of how status indices are defined, the direction of an edge is generally aligned with the difference in status between the endpoints.

The simplest example of a structural status index is indegree, which was generalized in numerous ways. Katz [Kat53], for instance, defines status by taking into account all directed walks ending at a vertex. the status of a vertex  $v \in V$  in a graph  $G = (V, E)$  is defined by  $p(v) = \sum_{u \in V} (\sum_{k=1}^{\infty} (\alpha A)^k)_{uv}$ , where  $1/\alpha$ ,  $0 < \alpha < 1$ , is an attenuation factor, and  $A$  the adjacency matrix of  $G$ . Recall that the entries  $(A^k)_{uv}$  give the number of walks of length  $k$  from vertex  $u$  to vertex  $v$ . Clearly, we obtain the same ranking as with indegree for very small  $\alpha$ , and attenuation must be large enough to make sure that the sum converges.

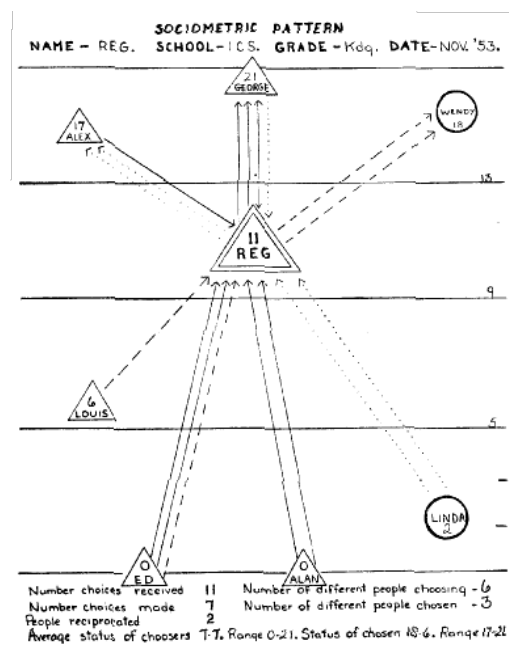
A natural class of layout algorithms that can be adapted for status drawings is the so-called Sugiyama framework, which is described in detail in Chapter ???. Its use was proposed in [BRW01], where the instantiation employs one-dimensional clustering of status scores for layer assignment and standard approaches for crossing reduction and horizontal coordinate assignment. Clustering is necessary since vertical coordinates are fixed and differences can be quite small, resulting in very close layers between which edges run almost horizontally. However, clustering worsens another, more general, open problem, namely how to accomodate intra-layer edges in layered layouts.

Two different approaches are less sensitive to this kind of problem and in addition more scalable. The first and simpler one is to fix  $y$ -coordinates and to determine the  $x$ -coordinates using a one-dimensional layout algorithm [BC03b], possibly taking the fixed dimension into account [KH05]. The second, more flexible, and likely to be more effective one is based on constrained optimization of stress using a gradient projection method [DKM09].



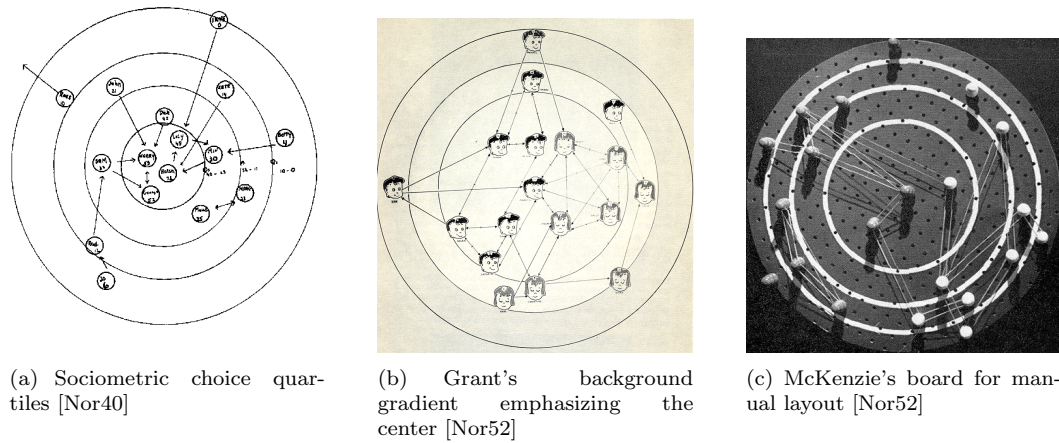


(a) extrinsically defined status [Why43]



(b) structurally defined status [Nor54]

**Figure 26.5** Two status diagrams using the high-low metaphor.



**Figure 26.6** Target diagrams in which rings correspond to levels of importance.

### Centrality

Similar to indices assessing status, centrality indices also have an immediate geometric connotation. A large value usually indicates that a vertex is structurally central, and a low value indicates that it is structurally peripheral. Just how strong, and sometimes confusing, the relationship between spatial metaphors and formal concepts can be is illustrated by the controversy of [Cha50, CJ51] about the structural status and location of prominent actors in the network of Figure 26.4(c).

It is therefore not surprising that centrality-based designs were proposed already in the 1940s. See Figure 26.6 for some historic examples.

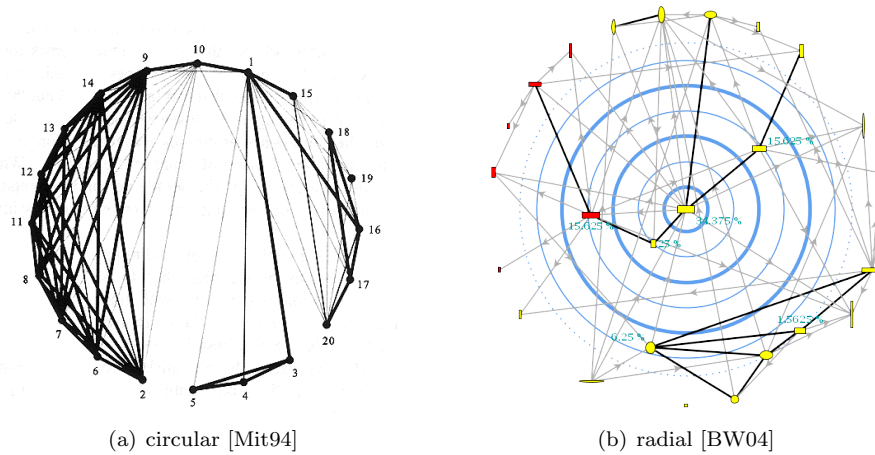
While the early designs were not based on an arbitrary index, but on indegree quartiles (representing four levels of prominence in *sociometric choice*), they are easily generalized to exact representation of any vertex centrality index  $c : V \rightarrow \mathbb{R}$ . The most frequently used indices are degree, closeness, and betweenness centrality [Fre79] as well as eigenvector centrality [Bon72].

Instead of placing vertices anywhere within one of four concentric rings, we can define their distance from the center of the drawing based on their centrality score, for instance using radii

$$r(v) = 1 - \frac{c(v) - \min_{x \in V} c(x)}{c_0 + \max_{x, y \in V} [c(x) - c(y)]},$$

as layout constraints [BKW03], where  $c_0$  is an offset creating space in the center and may depend on the number of highly central vertices. A constrained variant of the Kamada-Kawai approach (see Chapter ??) using polar coordinates for radial drawings is described in [Kam89]. In order to include crossings into the objective function, the method of [BKW03] is based on simulated annealing and, in addition, divided into phases in which more weight is placed on certain subconfigurations (related to confirmed and unconfirmed relationships).

Due to the radial coordinate constraints, crossings are not only a readability problem, but also an indication of poor angular distribution. An extreme example of this kind is presented in Figure 26.7(a), where a cutvertex and a separation pair are clearly visible by virtue of a circular ordering with few crossings. A second class of radial drawing algorithms is therefore based on combinatorial approaches that focus on crossing reduction in circular layouts to determine the angular coordinate (see Chapter ?? and [BB04]). More generally,



**Figure 26.7** Circular and radial drawing.

approaches for layered layout can be adapted to radial levels (Chapter ?? and [Bac07]).

The most recent approach uses stress minimization with a penalty term [BP11] measuring deviation from the assigned radii. By increasing the relative weight of the penalty term during iterative stress reduction, vertices are gradually forced to lie on their respective circle. While it appears that the less severe restrictions to which intermediate layouts are subjected may provide an advantage over gradient-projection methods, thorough experimentation is needed to determine which methods are most practical for centrality layouts.

To explore the differences between various centrality indices, methods extending the comparison based on scatterplot matrices [KS04] have been proposed in [DHK<sup>+</sup>06]. Among them is a force-directed method for the joint layout of stacked radial drawings of the same network with varying radial constraints. A visual variational approach to centrality within a network is introduced in [CCM12].

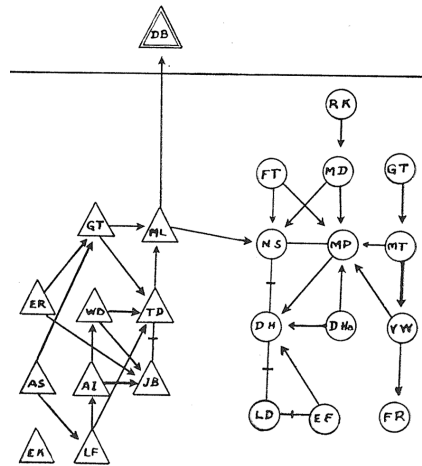
### 26.3.2 Cohesion

Cohesion is broadly defined as strong interconnectedness of a group of actors, although the formal structural definition of interconnectedness may vary according to type of relations and substantive interest. In the extreme, cohesion is defined in terms of cliques [LP49], but weaker definitions such as the degree-based cores [Sei83] or connectivity-based  $\lambda$ -sets [BES90] exist.

In Gestalt Theory [Wer44], the law of proximity suggests that cohesive, and in fact all types of groups can be represented effectively by placing their members closer to each other than to other actors. The friendship network of pupils in the 4th grade shown in Figure 26.8, for instance, is divided according to gender, and this striking correspondence between a vertex attribute and structural cohesion is made evident by spatial separation. However, there is little empirical evidence whether and which kind of cohesion is represented effectively by spatial proximity or separation in graphical representations of networks [MBK97, HHE07].

While the clumping of densely connected subgraphs is an implicit objective of force-directed and spectral layout algorithms (see Chapters ?? and ??), a layout algorithm should be generic and suitable for a range of cohesion measures.

In the previous section we assumed that the result of an analysis, a vertex index  $c : V \rightarrow \mathbb{R}$  representing prominence, is part of the input for a graph drawing algorithm. Similarly, let



**Figure 26.8** Friendship network of a 4th grade school class [Mor53, p. 163]. For graphical notation see Figure 26.2; note, though, the horizontal line delineating the class and the missing tick on the edge between NS and MP. The strong homophily effect is conveyed effectively through spatial separation.

us now assume that a cohesion analysis resulted in a decomposition that can be described in the following way.

A (*hierarchically*) *clustered graph*  $(G, T)$  is a graph  $G = (V, E)$  together with a rooted tree  $T$ , the *cluster tree*, such that the leaves of  $T$  correspond to a partition of  $V$  and each inner node is the union of the vertex sets of its children. Consequently, the root corresponds to the entire vertex set  $V$ . A clustered graph  $(G, T)$  is called *flat*, if  $T$  has height one, i.e. it is equivalent to a graph  $G = (V, E)$  together with a partition of  $V$ .

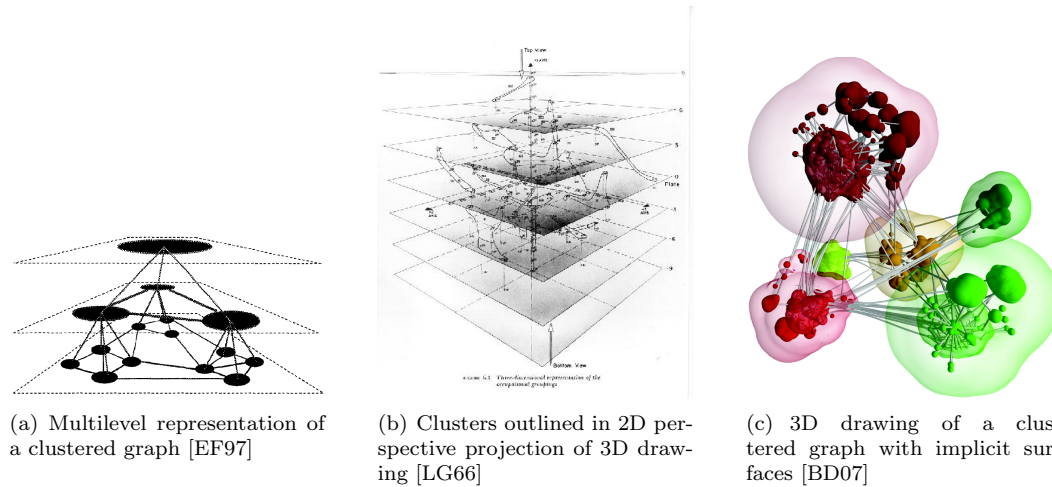
Note that cohesion analysis may result in other types of data. But one example are set covers of the vertices, which can be viewed as flat clustered graphs with overlapping clusters. They are equivalent to hypergraphs, which in turn are treated in Section 26.3.3.

### Clustered drawings

In an *inclusion drawing* of a (cluster) tree, vertices are represented as areas, and the parent-child relation is represented by area inclusion. A straightforward representation of clustered graphs consists of an inclusion drawing of the cluster tree overlaid on a drawing of the underlying graph such that vertices are inside their cluster boundaries and edges cross cluster boundaries at most once. Such a representation is called a *clustered drawing*, and at least topologically implements the idea that vertices of the same group belong together. Figures 26.1 and 26.9 provide examples.

Often, especially when the notion of cohesion and the implicit criteria of general layout algorithms coincide sufficiently well, clustered drawings are obtained by adding boundary curves to a layout obtained without consideration of the cluster tree. A typical example is the application of multidimensional scaling to a distance matrix with the addition of hierarchical clusters based on connectedness as shown in Figure 26.9(b).

Multidimensional scaling based on stress minimization and, in fact, all force-directed approaches can be customized to clustered graphs by adding cluster vertices that are connected to cluster members via short edges, and to other cluster vertices via long edges or even repulsion (e.g., [WM96, PNR08]). Alternatively, cohesion-based proximity can be ensured by a combination of space-filling and force-directed techniques that explicitly consider a cluster



**Figure 26.9** Clustered graphs.

tree [IMMS09]. Edge bundling along the cluster tree has been proposed as a method to reduce visual clutter [Hol06].

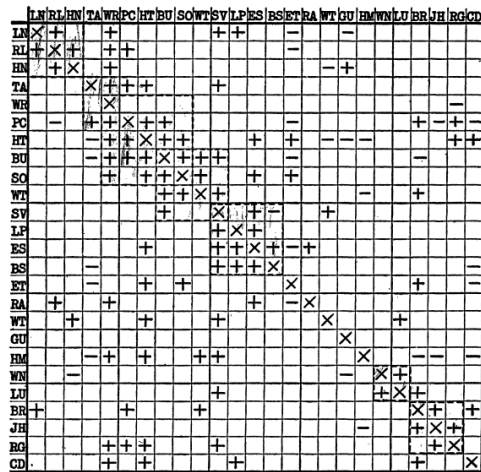
To avoid meaningless crossings, every edge should cross only boundaries of clusters on the unique path in the cluster tree that connects the leaves containing its endvertices. These crossings are called necessary. A clustered graph is called *c-planar* (*cluster planar*), if it can be drawn such that, simultaneously, there are no edge-edge crossings (i.e., the graph is planar) and there are no edge-region crossings (except those necessary) [EFN99]. The graph in Figure 26.1 is *c-planar*, although the drawing is not. Whether *c-planarity* can be tested efficiently is an interesting open problem [CDB05].

A conceptually different visualization approach is based on clustering via *semantic substrates* [SA06], where regions are prescribed for vertices belonging to an extrinsically defined cluster (most often by sharing selected attribute values), and layout is carried out using any method respecting region boundaries.

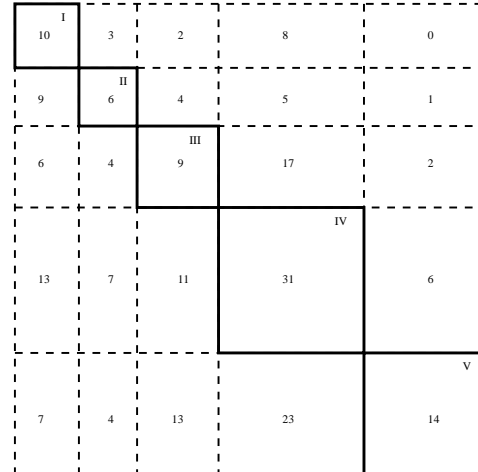
### Sociomatrices

In addition to the commonly used graph representation, there is also a tradition of depicting social relations in matrices. To distinguish them from sociograms, Moreno uses the term *sociomatrix* [Mor53]. Using the example in Figure 26.10 sociomatrices were advocated, e.g., in [FK46] (see also the interesting discussion that followed [Mor46, Kat47]), because they appear to be more effective at visualizing cohesion [GFC05]. Moreover, matrix cells are well-defined and compactly organized locations for information associated with the edges [vHSD09].

The main degree of freedom is the ordering of rows and columns, and its effect on visualization is illustrated in Figure 26.11. While Bertin [Ber83] appears to have coined the term *reorderable matrix* and reordering is already discussed in [FK46], the idea has been introduced much earlier [Pet99, Cze09]. Most relevant ordering problems are  $\mathcal{NP}$ -hard, though. They have been researched extensively under various names including *seriation* and *linear layout* [DPS02]. Often, the underlying ordering objectives aim at reducing the span of edges so that well-clustered graphs lead to visible blocks along the diagonal. For a clustered graph, an optimal ordering can be determined efficiently if the maximum degree of the cluster tree is bounded by a constant (see, e.g., [BDW99, Bra07]).

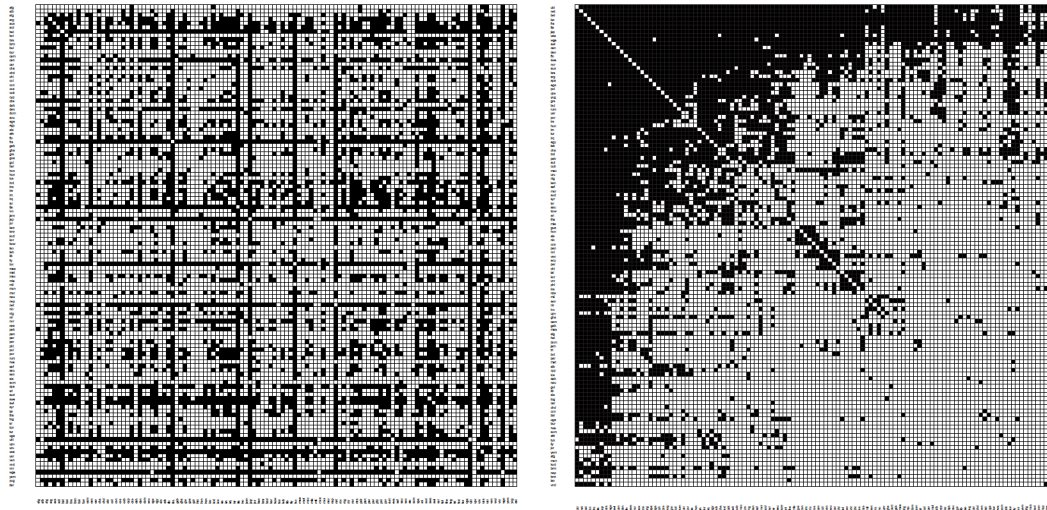


(a) ordered sociomatrix of a signed graph [FK46]

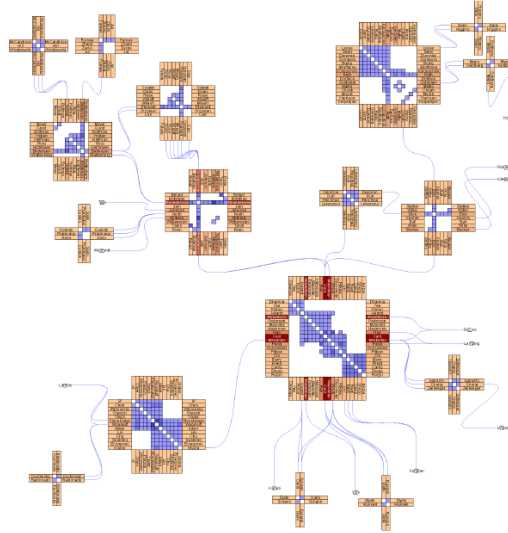


(b) blocked sociomatrix with edge counts [Lon48]

**Figure 26.10** Sociomatrix and block partition.



**Figure 26.11** Trade between countries reordered according to a hierachical clustering (reproduced from [BM04]).



**Figure 26.12** Integration of sociogram with sociomatrices for cohesive subgroups (reproduced from [HFM07]).

A system offering coordinated sociogram and sociomatrix views is MatrixExplorer [HF06]. The integrated view exemplified in Figure 26.12 uses both representations simultaneously and the decision which representation is used for which subgraph is based on the observation that matrix representations are especially suitable for dense (sub)graphs [HFM07]. Other augmentations of matrix representations to ease the recognition of paths include [HF07, SM07], and a matrix representation for layered graphs that has been applied to genealogies [BDF<sup>+</sup>10].

An integrated representation that is not based on a matrix of adjacencies, but a grid layout of vertex attribute levels, are PivotGraphs [Wat06]. They generalize attribute-defined layouts (cf. Figure 26.4(d)) and are particularly suited for the interactive exploration of associations between vertex attributes and edges.

### 26.3.3 Two-mode networks

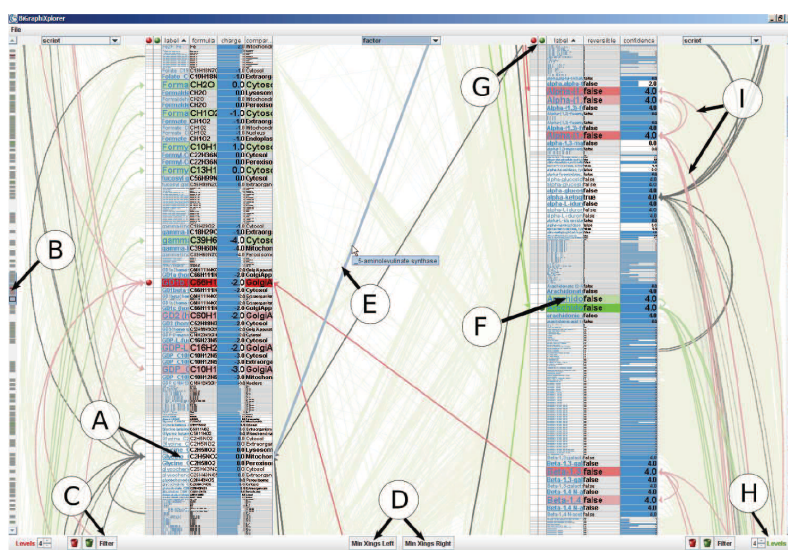
The networks considered so far are actually *one-mode* networks, because their vertices represent elements of the same mode or category such as persons. Quite frequently, however, the relation of interest is between elements of different categories such as persons and groups [Bre74]. Such networks can be represented in rectangular matrices with rows and columns indexed by the respective categories, and they are referred to as *two-mode* networks.

Two-mode networks are often visualized like one-mode networks, with different appearances for vertices from the two categories. However, their distinctive characteristic of being bipartite with a prescribed bipartition of vertices can also guide a layout algorithm.

As a variant of spectral layout for one-mode networks, left and right singular vectors of the rectangular adjacency matrix (or other matrices derived from it) can be used for coordinates. An entire family of related techniques is reviewed in [dLM00]. See also [Bre09] for a closely related analytic technique and [WG98] for a comparison with graphical representations described in the following subsection.

More combinatorial approaches are those developed for bipartite graphs. These include, in particular, drawings in which the vertices of the two modes are placed on different parallel





**Figure 26.13** A two-mode network with straight-line edges drawn between attribute tables of the two node sets, and its one-mode projections drawn with curved edges on the sides (reproduced from [SJUS08]).

lines (2-level drawings) or, more generally, in separable regions. As usual, graph drawing research in this area has focused on conditions under which the resulting drawings can be made planar [Bie98, BKM98, CSW04, DGGL08] and on the difficulty of crossing minimization [ZSE05]. It would be interesting to identify criteria for informative visualization of bipartite graphs.

An example of several methods combining ideas of spatial separation and relative placement are the anchored maps of [Mis07]. While one set of the bipartition is arranged on a circle, vertices in the other are placed relative to their neighbors. An interesting mixture of 2-layer drawing and tabular representation is exemplified in Figure 26.13.

### Hypergraphs

If it makes sense to consider the elements of one of the two modes as subsets of the other (as with groups and persons such as company boards and directors), a two-mode network can be treated as a hypergraph. In addition to those associated with the above bipartite graph model, several other graphical representations are available.

A straightforward variant is the *edge standard*, which is based on an ordinary layout of the bipartite graph representation of the hypergraph, but with a different rendering of the induced star subgraphs that represent hyperedges. This star may be substituted for a tree to shorten the total length of the hyperedge. For directed hypergraphs, layout constraints can be used to enable directed edges to be rendered confluent [Mäk90].

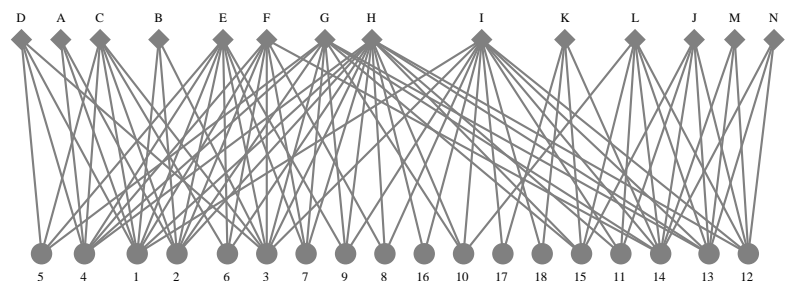
*Subdivision drawings* [KvKS09] are subdivisions of the plane such that each vertex corresponds to a region and the set of regions corresponding to a hyperedge is connected. This requires that the hypergraph has a planar *support*, i.e. the existence of a planar graph on the same vertices such that each of the hyperedges of the original hypergraph induces a connected subgraph. Deciding whether a hypergraph has a planar support is  $\mathcal{NP}$ -complete [JP87]. Tree supports, on the other hand, are characterized by the existence of an elimination ordering in which vertices contained in only one hyperedge, or in a subset of the hyperedges containing some other vertex, are removed iteratively. The main open



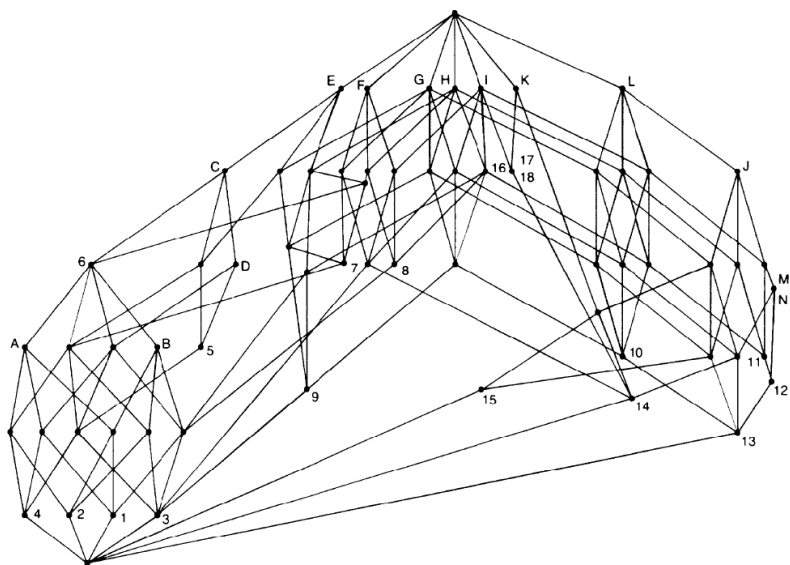


NAMES OF PARTICIPANTS OR GROUP I	CODE NUMBERS AND DATES OF SOCIAL EVENTS REPORTED IN <i>Old City Herald</i>													
	(1) 6/22	(2) 3/2	(3) 4/12	(4) 9/26	(5) 2/25	(6) 5/19	(7) 3/15	(8) 9/16	(9) 4/8	(10) 6/10	(11) 2/23	(12) 4/7	(13) 11/21	(14) 3/3
1. Mrs. Evelyn Jefferson.....	X	X	X	X	X	X	X	X	X					
2. Miss Laura Mandeville.....	X	X	X	X	X	X	X	X	X					
3. Miss Theresa Anderson.....	X	X	X	X	X	X	X	X	X					
4. Miss Brenda Rogers.....	X		X	X	X	X	X	X						
5. Miss Charlotte McDowd.....			X	X	X	X	X							
6. Miss Frances Anderson.....			X		X	X	X	X						
7. Miss Eleanor Nye.....					X	X	X	X						
8. Miss Pearl Oglethorpe.....					X	X	X	X	X					
9. Miss Ruth DeSand.....					X		X	X	X					
10. Miss Verne Sanderson.....						X	X	X	X			X		
11. Miss Myra Liddell.....							X	X	X	X		X		
12. Miss Katherine Rogers.....							X	X	X	X		X	X	X
13. Mrs. Sylvia Avondale.....							X	X	X	X		X	X	X
14. Mrs. Nora Fayette.....						X		X	X	X	X	X	X	X
15. Mrs. Helen Lloyd.....							X	X	X	X	X	X		
16. Mrs. Dorothy Murchison.....								X	X	X				
17. Mrs. Olivia Carleton.....								X	X	X	X			
18. Mrs. Flora Price.....								X	X	X	X			

(a) data matrix [DGG41]



(b) two-layer drawing of bipartite network



(c) lattice representation [FW93]

**Figure 26.15** The Southern Women data of Davis, Gardner and Gardner [DGG41] is a two-mode network of 18 women's (labeled 1–18) attendance of 14 events (labeled A–N).

corresponding women. Nodes are ordered by set-inclusion, i.e. for each node the set of women consists of those encountered on downward paths to the bottom, and the set of events consists of those encountered on upward paths to the top. A study essentially concluded that users do understand such diagrams [EDB04].

Approaches for drawing lattices range from layer-constrained force-directed layout in three dimensions [Fre04b] to enumeration and decomposition approaches [RCE06, BPS11]. See the review in [MH01] for relations with other combinatorial structures.

### 26.3.4 Dynamics

Temporal aspects are considered explicitly in *longitudinal* social network analysis. Mostly, this is concerned with panel data, i.e., cross-sectional states of networks observed at discrete time points. This is immediate when data is collected in waves. Even when the situation is more accurately described in terms of relational events (such as phone calls), however, these are often aggregated over time intervals into cross-sectional graph representations to ensure applicability of the wide range of methods developed for (static) graphs.

Inbetween observations or aggregations, the sets of vertices and edges, as well as attribute values may be subject to change. Typical research questions are: how do actor characteristics effect structural change (*social selection*), and how do structural conditions effect actor behavior (*social influence*)? In addition, the subject of interest may actually be a process such as the diffusion of information taking place on a (possibly changing) network.

By combinatorial explosion, this leads to numerous problem variants. The variant most extensively researched in graph drawing, however, focuses on *dynamic graphs* which consist of a sequence of interrelated graphs  $G^{(1)}, \dots, G^{(T)}$ , called *states*. In social network analysis these arise from panel data on social structure (network evolution). Research on a *streaming* scenario in which a single graph becomes available one edge at a time was initiated only recently [BBDB<sup>+</sup>10], but may soon become relevant for dyadic event data.

There are two main scenarios for visualizing dynamic graphs, *online* and *offline* dynamic graph drawing. Layout approaches for these are considered in more detail below. In both cases, a solution consists of a sequence of layouts, one for each  $G^{(t)}$  with  $t = 1, \dots, T$ , and two conflicting criteria are used to evaluate the quality of a solution.

On the one hand, each layout in the sequence should be acceptable with respect to the criteria of a static graph drawing problem. We refer to this requirement as *layout quality*, and assume that the related static layout problem is fixed. On the other hand, the degree of change between consecutive layouts should be indicative of the degree of change between the corresponding graphs. This criterion is referred to as *layout stability* and generally motivated by preservation of a user's mental map [MELS95].

Note that the stability requirement applies to the difference between consecutive layouts and, depending on the visualization media, also to the transition from one layout to the next. These two aspects are referred to as the *logical* and the *physical* update, respectively [Nor96]. Difference metrics for pairs of layouts are treated in [BT00], and animation between layouts is the subject of [FE02]. The interpolation approach of [BFP07] implements the physical update as a refinement of the logical update.

#### Online Scenario

In an *online* scenario, a dynamic graph is presented one state at a time, and the layout of a state is to be determined before the next state is known. Stability can therefore only be introduced with respect to layouts of previous states.

Since iterative layout algorithms are very common in applied graph drawing in gen-

eral, and social network visualization in particular, a simple approach to online drawing is the initialization of the layout algorithm for a state with the layout of the previous state [MMBd05, HEW98]. This is a convenient, though indirect, approach to address the stability requirement. It does work fairly well when the iterations of the layout algorithm are used for the physical updates of an animation.

Explicit consideration of stability by defining a layout objective that trades off quality and stability using a static objective and a difference metric is proposed in the Bayesian framework of [BW97]. A recent application of this is [FT08]. Since the employed difference metric is penalizing vertex movement via (weighted) distances from previous positions, this is a case of what is called anchoring [LMR98].

An unusual variant of online drawing is introduced in [DDBF<sup>+</sup>99], where drawings of a state must keep the drawing of the previous state intact, and expectations (rather than knowledge) about the additional subgraph to accommodate are available. However, this situation has so far been considered only for descending traversals of trees.

### Offline Scenario

In an *offline* scenario, the input given is a dynamic graph and the output sought is a layout for each of its states. Except for streaming event data and some applications involving social networking sites, the offline scenario is the typical scenario in empirical social network analysis. Since the entire sequence of states is known before any layout needs to be determined, the layout of a state may be determined with subsequent states in mind. In other words, we may use knowledge about the future.

Note that methods for online scenarios can be applied in offline scenarios, although at the possible expense of both quality and stability, but the reverse is generally not possible.

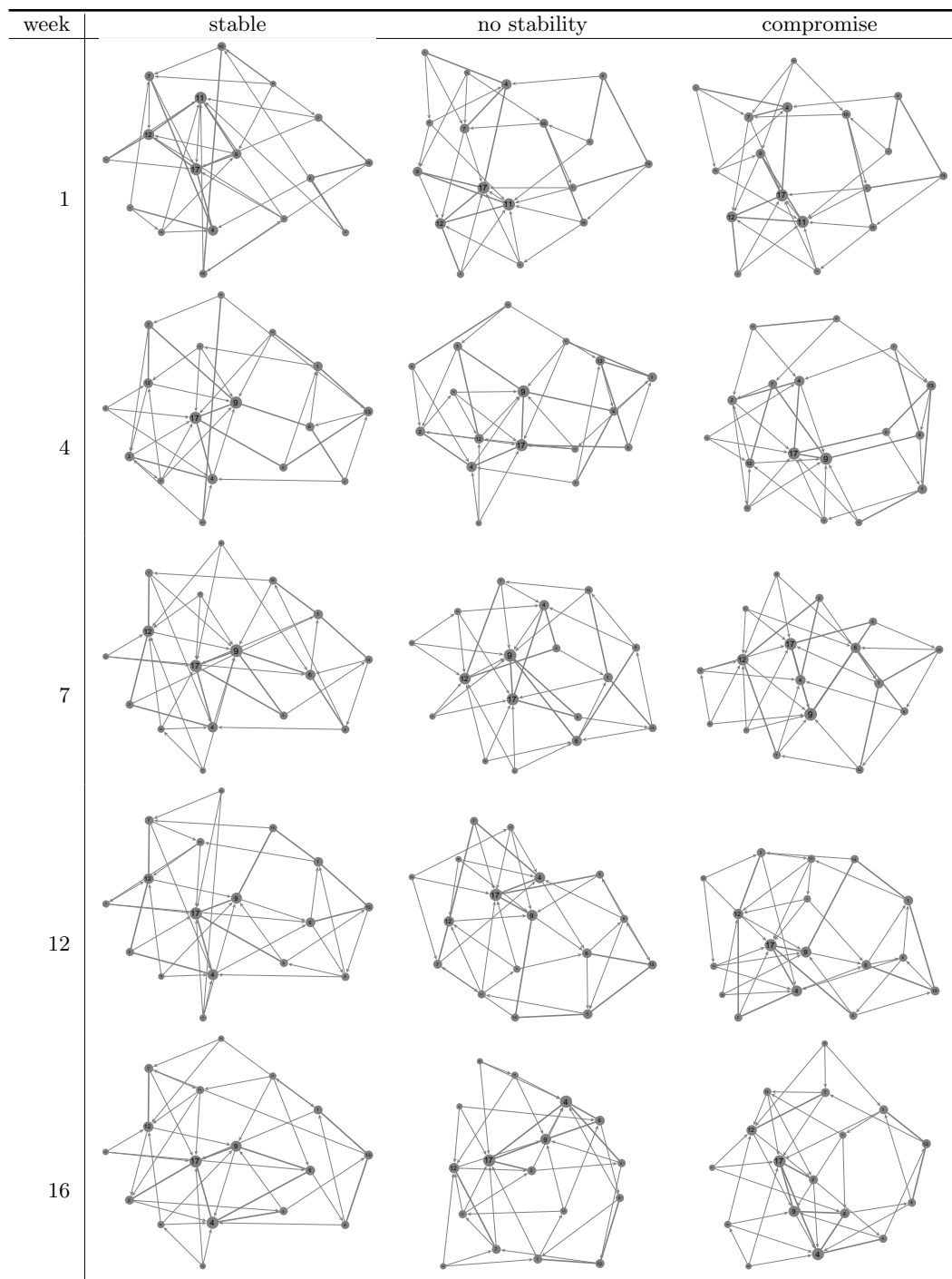
In a recent review [BIM12], three primary approaches to offline dynamic graph drawing are distinguished:

**Aggregation.** All graphs in the sequence are aggregated into a single graph that has one vertex for each actor. The position of each occurrence of vertex in a state is fixed by the layout of the aggregated graph. Variants of this approach are considered, e.g., in [BC03a, DG04, MMBd05], and it is referred to as the *flip-book approach* in the last reference.

**Anchoring.** Using auxiliary edges, vertices are connected to immobile copies fixed to a desired location which may be, for instance, the previous position as in an online scenario, or a reference position determined from an aggregate layout in an offline scenario. This approach is used, e.g., in [LMR98, BW97, FT08].

**Linking.** All graphs in the sequence are combined into a single graph that has one vertex for each occurrence of an actor, and an edge is created between vertices representing the same actor in consecutive graphs. A layout of this graph directly yields positions for all vertex instances in the sequence. This approach is used, e.g., in [DG04, EKLN04, DHK<sup>+</sup>06].

Algorithmic experimental evidence [BM12] suggests that, at least for methods based on stress minimization [GKN04] and general conditions, linking dominates anchoring in terms of stability and quality. On the other hand, anchoring is computationally cheaper and especially suited when a dynamic graph has rather persistent global structure. Evidence from user experiments, on the other hand, is inconclusive about the actual value of stability in dynamic graph animation [PS08], and even animation itself [APP11].



**Figure 26.16** Five states of the dynamic graph obtained from top-3 choices in Newcomb's fraternity data. Layout obtained by stress minimization on an aggregate graph (stable), initialized by previous layout (no stability), and with linking of consecutive layouts (compromise).



**Figure 26.17** Gestaltmatrix of Newcomb's fraternity data [BN11]. Matrix cells show evolution of rank (length) and balance (angle) of pairwise nominations. Color on diagonal indicates average deviation of received nominations from expected value.

Online and offline approaches can be compared using the small multiples in Figure 26.16. The networks shown form a subset of the famous Newcomb fraternity data [New61]. The full data are shown in Figure 26.17 using a static matrix-based representation for dynamic directed graphs that have a numerical edge attribute [BN11].

## 26.4 Trends and Challenges

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While graphs arising from social networks across application domains exhibit some general tendencies such as sparseness and local clustering, there is no formal characterization in terms of structural requirements that delineates this class of graphs from others. Likewise, the information to be conveyed in a social network visualization differs by available data, interest, and domain.

Layout of social networks is therefore contingent on many factors, and comparison of approaches is possible only if scope and purpose are defined precisely. Identification of practically relevant and pragmatic tasks remains a challenge, though.

Despite the range of approaches presented in this chapter, force-directed methods – much like in other areas of applied graph drawing – are most commonly used for social network layout. This is likely because of their generality, simplicity, adaptability, and above all their availability. While force-directed methods generally perform well in separating clusters in graphs with varying local density, these methods are particularly troubled by small distances and skewed degree distributions [BP09]. A fundamental challenge is therefore to identify representations and layout criteria that allow to deal with such structures [ACL07].

Another very general challenge involves the interplay between methods for hierarchical clustering of graphs and clustered graph layout. Especially for large graphs, hierarchical clustering is frequently used as a tool in multilevel layout algorithms, but the artifacts resulting from the choice of clustering or filter methods are not yet understood well (see, e.g., [JHGH08, vHW08, HN07]).

Two research directions that are more closely related to the type of data (rather than its properties) encountered in social network analysis are the genuine treatment of two-mode networks (Section 26.3.3) and visual means to support stochastic network modeling [BIM12].

Finally, there is also at least one example where artistic drawings of social networks inspired a new graph drawing convention, namely Lombardi drawings [Hob03, DEG<sup>+</sup>12].

Many software packages are available already for the analysis and visualization of social networks, and many more are introduced for specific application domains. Among social scientists, UCINET [BEF99] is the most widely known. At the time of this writing, Pajek [BM04] is likely to be the most widely used across all disciplines, and visone [BW04] the social network analysis tool with the most sophisticated graph drawing features. Other comprehensive and popular tools include Tulip [Aub04], NodeXL [HSS10], Gephi [BHJ09], and ORA [CRSC11]. A recent software review can be found in [HvD11], and a comparative evaluation of some tools is attempted in [XTT<sup>+</sup>10]. A comprehensive list of software for social networks is maintained in Wikipedia.<sup>1</sup>

Very likely the most dominant force driving visualization research on social networks over the next decade will be online social and other networks derived from social media [Fur10]. This is in part because they combine virtually all the current challenges of size and dynamics with the more specific challenges that arise from multivariate complexity. Moreover, such research can be of economic relevance, draw large audiences, and make use of easily accessible data [LPA<sup>+</sup>09]. It will be exciting to witness whether graph drawing can make significant contributions to this area and thus challenge currently reigning adaptations of its oldest methods (see [HB05] for one solid example). An example in this direction is a layout algorithm for digital social networks tailored to smartphone displays [DLDBI12].

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<sup>1</sup>[http://en.wikipedia.org/wiki/Social\\_network\\_analysis\\_software](http://en.wikipedia.org/wiki/Social_network_analysis_software)

# Bibliography

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- [ACL07] Reid Andersen, Fan Chung, and Linyuan Lu. Drawing power law graphs using a local/global decomposition. *Algorithmica*, 47(4):379–397, 2007.
- [APP11] Daniel Archambault, Helen C. Purchase, and Bruno Pinaud. Animation, small multiples, and the effect of mental map preservation in dynamic graphs. *IEEE Transactions on Visualization and Computer Graphics*, 17(4):539–552, 2011.
- [ARRC11] Basak Alper, Nathalie Riche, Gonzalo Ramos, and Mary Czerwinski. Design study of linesets, a novel set visualization technique. *IEEE Transactions on Visualization and Computer Graphics*, 17(12):2259–2267, 2011.
- [Aub04] David Auber. Tulip – a huge graph visualisation framework. In Jünger and Mutzel [JM04], pages 105–126.
- [Bac07] Christian Bachmaier. A radial adaptation of the Sugiyama framework for visualizing hierarchical information. *IEEE Transactions on Visualization and Computer Graphics*, 13(3):583–594, 2007.
- [Bar54] John A. Barnes. Class and committees in a Norwegian island parish. *Human Relations*, 7:39–58, 1954.
- [BB04] Michael Baur and Ulrik Brandes. Crossing reduction in circular layouts. In Juraj Hromkovič, Manfred Nagl, and Bernhard Westfechtel, editors, *Proceedings of the 30th International Workshop on Graph-Theoretical Concepts in Computer Science (WG'04)*, volume 3353 of *Lecture Notes in Computer Science*, pages 332–343. Springer-Verlag, 2004.
- [BBDB<sup>+</sup>10] Carla Binucci, Ulrik Brandes, Giuseppe Di Battista, Walter Didimo, Marco Gaertler, Pietro Palladino, Maurizio Patrignani, Antonios Symvonis, and Katharina A. Zweig. Drawing trees in a streaming model. In *Proceedings of the 17th International Symposium on Graph Drawing (GD'09)*, volume 5849 of *Lecture Notes in Computer Science*, pages 292–303. Springer-Verlag, 2010.
- [BC03a] Ulrik Brandes and Steven R. Corman. Visual unrolling of network evolution and the analysis of dynamic discourse. *Information Visualization*, 2(1):40–50, 2003.
- [BC03b] Ulrik Brandes and Sabine Cornelsen. Visual ranking of link structures. *Journal of Graph Algorithms and Applications*, 7(2):181–201, 2003.
- [BCPS11a] Ulrik Brandes, Sabine Cornelsen, Barbara Pampel, and Arnaud Sallaberry. Blocks of hypergraphs (applied to hypergraphs and outerplanarity). In *Proceedings of the 21st International Workshop on Combinatorial Algorithms (IWOCA 2010)*, volume 6460 of *Lecture Notes in Computer Science*, pages 201–211. Springer-Verlag, 2011.
- [BCPS11b] Ulrik Brandes, Sabine Cornelsen, Barbara Pampel, and Arnaud Sallaberry. Path-based supports for hypergraphs. In *Proceedings of the 21st International Workshop on Combinatorial Algorithms (IWOCA 2010)*, volume 6460 of *Lecture Notes in Computer Science*, pages 20–33. Springer-Verlag, 2011.
- [BD07] Michael Balzer and Oliver Deussen. Level-of-detail visualization of clustered graph layouts. In *Proceedings of the 6th International Asia-Pacific Symposium on Visualisation (APVis '07)*, pages 133–140. IEEE, 2007.



- [BDF<sup>+</sup>10] Anastasia Bezerianos, Pierre Dragicevic, Jean-Daniel Fekete, Juhee Bae, and Ben Watson. GeneaQuilts: A system for exploring large genealogies. *IEEE Transactions on Visualization and Computer Graphics*, 16(6):741–748, 2010.
- [BDW99] Rainer Burkhard, Vladimir G. Deĭneko, and Gerhard Woeginger. The travelling salesman and the PQ-tree. *Mathematics of Operations Research*, 24(1):262–272, 1999.
- [BE00] François Bertault and Peter Eades. Drawing hypergraphs in the subset standard. In *Proceedings of the 8th International Symposium on Graph Drawing (GD’00)*, volume 1984 of *Lecture Notes in Computer Science*, pages 164–169. Springer-Verlag, 2000.
- [BE05] Ulrik Brandes and Thomas Erlebach, editors. *Network Analysis: Methodological Foundations*, volume 3418 of *Lecture Notes in Computer Science*. Springer-Verlag, 2005.
- [BE06] Stephen P. Borgatti and Martin G. Everett. A graph-theoretic perspective on centrality. *Social Networks*, 28:466–484, 2006.
- [BEF99] Stephen P. Borgatti, Martin G. Everett, and Linton C. Freeman. *UCINET 6.0 Version 1.00*. Analytic Technologies, 1999.
- [Ber83] Jacques Bertin. *Semiology of Graphics: Diagrams, Networks, Maps*. University of Wisconsin Press, 1983.
- [BES90] Stephen P. Borgatti, Martin G. Everett, and Paul R. Shirey. LS sets, lambda sets and other cohesive subsets. *Social Networks*, 12(4):337–357, 1990.
- [BFP07] Ulrik Brandes, Daniel Fleischer, and Thomas Puppe. Dynamic spectral layout with an application to small worlds. *Journal of Graph Algorithms and Applications*, 11(2):325–343, 2007.
- [BHJ09] Mathieu Bastian, Sebastien Heymann, and Mathieu Jacomy. Gephi: an open source software for exploring and manipulating networks. In *Proceedings of the 3rd International AAAI Conference on Weblogs and Social Media (ICWSM ’09)*, pages 361–362, 2009.
- [Bie98] Therese C. Biedl. Drawing planar partitions I: LL-drawings and LH-drawings. In *Proceedings of the 14th Annual ACM Symposium on Computational Geometry (SoCG’98)*, pages 287–296, 1998.
- [BIM12] Ulrik Brandes, Natalie Indlekofer, and Martin Mader. Visualization methods for longitudinal social networks and stochastic actor-oriented modeling. *Social Networks*, 34(3):291–308, 2012.
- [BKM98] Therese C. Biedl, Michael Kaufmann, and Petra Mutzel. Drawing planar partitions II: HH-drawings. In *Proceedings of the 24th International Workshop on Graph-Theoretical Concepts in Computer Science (WG’98)*, volume 1517 of *Lecture Notes in Computer Science*, pages 101–114. Springer-Verlag, 1998.
- [BKR<sup>+</sup>99] Ulrik Brandes, Patrick Kenis, Jörg Raab, Volker Schneider, and Dorothea Wagner. Explorations into the visualization of policy networks. *Journal of Theoretical Politics*, 11(1):75–106, 1999. Reprinted in Linton Freeman, ed., *Social Network Analysis*, vol. I (Data, Mathematical Models, and Graphics), Sage, 2007.
- [BKR06] Ulrik Brandes, Patrick Kenis, and Jörg Raab. Explanation through network visualization. *Methodology*, 2(1):16–23, 2006. Spanish translation in *REDES* 9(6), 2005.
- [BKW03] Ulrik Brandes, Patrick Kenis, and Dorothea Wagner. Communicating centrality in policy network drawings. *IEEE Transactions on Visualization and Computer Graphics*, 9(2):241–253, 2003.

- [BM04] Vladimir Batagelj and Andrej Mrvar. Pajek – analysis and visualization of large networks. In Jünger and Mutzel [JM04], pages 77–103.
- [BM12] Ulrik Brandes and Martin Mader. A quantitative comparison of stress-minimization approaches for offline dynamic graph drawing. In *Proceedings of the 19th International Symposium on Graph Drawing (GD 2011)*, volume 7034 of *Lecture Notes in Computer Science*, pages 99–110. Springer-Verlag, 2012.
- [BMBL09] Stephen P. Borgatti, Ajay Mehra, Daniel J. Brass, and Giuseppe Labianca. Network analysis in the social sciences. *Science*, 323(5916):892–895, 2009.
- [BN11] Ulrik Brandes and Bobo Nick. Asymmetric relations in longitudinal social networks. *IEEE Transactions on Visualization and Computer Graphics*, 17(12):2283–2290, 2011.
- [Bon72] Phillip Bonacich. Factoring and weighting approaches to status scores and clique identification. *Journal of Mathematical Sociology*, 2:113–120, 1972.
- [Bor05] Stephen P. Borgatti. Centrality and network flow. *Social Networks*, 27:55–71, 2005.
- [BP09] Ulrik Brandes and Christian Pich. An experimental study on distance-based graph drawing. In *Proceedings of the 16th International Symposium on Graph Drawing (GD’08)*, volume 5417 of *Lecture Notes in Computer Science*, pages 218–229. Springer-Verlag, 2009.
- [BP11] Ulrik Brandes and Christian Pich. More flexible radial layout. *Journal of Graph Algorithms and Applications*, 15(1):157–173, 2011.
- [BPS11] Anne Berry, Romain Pogorelnik, and Alain Sigayret. Vertical decomposition of a lattice using clique separators. In *Proceedings of the 8th International Conference on Concept Lattices and their Applications (CLA’11)*, pages 15–29, 2011.
- [Bra96] Franz J. Brandenburg, editor. *Proceedings of the 3rd International Symposium on Graph Drawing (GD ’95)*, volume 1027 of *Lecture Notes in Computer Science*. Springer, 1996.
- [Bra07] Ulrik Brandes. Optimal leaf ordering of complete binary trees. *Journal of Discrete Algorithms*, 5(3):546–552, 2007.
- [Bra08] Ulrik Brandes. Social network analysis and visualization. *IEEE Signal Processing Magazine*, 25(6):147–151, 2008.
- [Bre74] Ronald L. Breiger. The duality of persons and groups. *Social Forces*, 53(2):181–190, 1974.
- [Bre09] Ronald L. Breiger. On the duality of cases and variables. In David Byrne and Charles C. Ragin, editors, *The SAGE Handbook of Case-Based Methods*, pages 243–259. Sage, 2009.
- [BRW01] Ulrik Brandes, Jörg Raab, and Dorothea Wagner. Exploratory network visualization: Simultaneous display of actor status and connections. *Journal of Social Structure*, 2(4), 2001.
- [BT00] Stina S. Bridgeman and Roberto Tamassia. Difference metrics for interactive orthogonal graph drawing algorithms. *Journal of Graph Algorithms and Applications*, 4(3):47–74, 2000.
- [But09] Carter T. Butts. Revisiting the foundations of network analysis. *Science*, 325(5939):414–416, 2009.
- [BvKM<sup>+</sup>10] Kevin Buchin, Marc van Kreveld, Henk Meijer, Bettina Speckmann, and Kevin Verbeek. On planar supports or hypergraphs. In *Proceedings of the 17th International Symposium on Graph Drawing (GD’09)*, volume 5849 of *Lecture Notes in Computer Science*, pages 345–356. Springer-Verlag, 2010.

- [BW97] Ulrik Brandes and Dorothea Wagner. A Bayesian paradigm for dynamic graph layout. In Giuseppe Di Battista, editor, *Proceedings of the 5th International Symposium on Graph Drawing (GD '97)*, volume 1353 of *Lecture Notes in Computer Science*, pages 236–247. Springer, 1997.
- [BW04] Ulrik Brandes and Dorothea Wagner. visone – analysis and visualization of social networks. In Michael Jünger and Petra Mutzel, editors, *Graph Drawing Software*, pages 321–340. Springer-Verlag, 2004.
- [CAI] CAIDA. Cooperative association for internet data analysis. [http://www.caida.org/research/topology/as\\_core\\_network/](http://www.caida.org/research/topology/as_core_network/).
- [CCM12] Carlos D. Correa, Tarik Crnovrsanin, and Kwan-Liu Ma. Visual reasoning about social networks using centrality sensitivity. *IEEE Transactions on Visualization and Computer Graphics*, 18(1):106–120, 2012.
- [CDB05] Pier Francesco Cortese and Giuseppe Di Battista. Clustered planarity. In *Proceedings of the 21st Annual ACM Symposium on Computational Geometry (SoCG'05)*, pages 32–34, 2005.
- [Cha50] F. Stuart Chapin. Sociometric stars as isolates. *American Journal of Sociology*, 56(3):263–267, 1950.
- [CJ51] Joan H. Criswell and Helen Hall Jennings. A critique of chapin’s ‘sociometric stars as isolates’. *American Journal of Sociology*, 57(3):260–264, 1951.
- [CM11] Carlos D. Correa and Kwan-Liu Ma. Visualizing social networks. In Charu C. Aggarwal, editor, *Social Network Data Analytics*, pages 307–326. Springer-Verlag, 2011.
- [CPC09] Christopher Collins, Gerald Penn, and Sheelagh Carpendale. Bubble sets: revealing set relations with isocontours over existing visualizations. *IEEE Transactions on Visualization and Computer Graphics*, 15(6):1009–1016, 2009.
- [CRSC11] Kathleen M. Carley, Jeff Reminga, Jon Storrick, and Dave Columbus. ORA user’s guide 2011. Technical Report CMU-ISR-11-107, Carnegie Mellon University, Institute for Software Research, 2011.
- [CSW04] Sabine Cornelsen, Thomas Schank, and Dorothea Wagner. Drawing graphs on two and three lines. *Journal of Graph Algorithms and Applications*, 8(2):161–177, 2004.
- [CSW05] Peter J. Carrington, John Scott, and Stanley Wasserman, editors. *Models and Methods in Social Network Analysis*. Cambridge University Press, 2005.
- [CY10] Ing-Xiang Chen and Cheng-Zen Yang. Visualization of social networks. In *Handbook of Social Network Technologies and Applications* [Fur10], pages 585–610.
- [Cze09] Jan Czekanowski. Zur Differentialdiagnose der Neandertalgruppe. *Korrespondenz-Blatt der Deutschen Gesellschaft für Anthropologie, Ethnologie und Urgeschichte*, 40(6/7):44–47, 1909.
- [DBF05] Patrick Doreian, Vladimir Batagelj, and Anuška Ferligoj. *Generalized Block-modeling*. Cambridge University Press, 2005.
- [DDBF<sup>+</sup>99] Camil Demetrescu, Giuseppe Di Battista, Irene Finocchi, Giuseppe Liotta, Maurizio Patrignani, and Maurizio Pizzonia. Infinite trees and the future. In *Proceedings of the 7th International Symposium on Graph Drawing (GD'99)*, volume 1731 of *Lecture Notes in Computer Science*, pages 379–391. Springer-Verlag, 1999.
- [DEG<sup>+</sup>12] Christian A. Duncan, David Eppstein, Michael T. Goodrich, Stephen G. Kobourov, and Martin Nöllenburg. Lombardi drawings of graphs. *Journal of Graph Algorithms and Applications*, 16(1):85–108, 2012.

- [DG04] Tim Dwyer and David R. Gallagher. Visualising changes in fund manager holdings in two and a half-dimensions. *Information Visualization*, 3(4):227–244, 2004.
- [DGG41] Allison Davis, Burleigh B. Gardner, and Mary R. Gardner. *Deep South: A Social Anthropological Study of Caste and Class*. University of Chicago Press, 1941.
- [DGGL08] Emilio Di Giacomo, Luca Grilli, and Giuseppe Liotta. Drawing bipartite graphs on two parallel convex curves. *Journal of Graph Algorithms and Applications*, 12(1):97–112, 2008.
- [DGL08] Walter Didimo, Francesco Giordano, and Giuseppe Liotta. Overlapping cluster planarity. *Journal of Graph Algorithms and Applications*, 12(3):267–291, 2008.
- [DHK<sup>+</sup>06] Tim Dwyer, Seokhee Hong, Dirk Koschützki, Falk Schreiber, and Kai Xu. Visual analysis of network centralities. In *Proceedings of the 2006 Asia-Pacific Symposium on Information Visualisation (APVis '06)*, pages 189–197. Australian Computer Society, 2006.
- [DKM09] Tim Dwyer, Yehuda Koren, and Kim Marriott. Constrained graph layout by stress majorization and gradient projection. *Discrete Mathematics*, 309(7):1895–1908, 2009.
- [DLDB12] Giordano Da Lozzo, Giuseppe Di Battista, and Francesco Ingrassia. Drawing graphs on a smartphone. *Journal of Graph Algorithms and Applications*, 16(1):109–126, 2012.
- [dLM00] Jan de Leeuw and George Michailides. Graph layout techniques and multivariate data analysis. In F. Thomas Bruss and Lucien Le Cam, editors, *Game theory, optimal stopping, probability and statistics: Papers in honor of Thomas S. Ferguson*, pages 219–248. Beachwood, 2000.
- [DPS02] Josep Diaz, Jordi Petit, and Maria Serna. A survey of graph layout problems. *ACM Computing Surveys*, 34:313–356, 2002.
- [Duq99] Vincent Duquenne. Latticial structures in data analysis. *Theoretical Computer Science*, 217:407–436, 1999.
- [EDB04] Peter Eklund, Jon Ducrou, and Peter Brawn. Concept lattices for information visualization: Can novices read line-diagrams? In *Proceedings of the 2nd International Conference on Formal Concept Analysis (ICFCA'04)*, volume 2961 of *Lecture Notes in Computer Science*, pages 235–236. Springer-Verlag, 2004.
- [EF97] Peter Eades and Qing-Wen Feng. Multilevel visualization of clustered graphs. In *Proceedings of the 5th International Symposium on Graph Drawing (GD'97)*, volume 1190 of *Lecture Notes in Computer Science*, pages 101–112. Springer-Verlag, 1997.
- [EFN99] Peter Eades, Qing-Wen Feng, and Hiroshi Nagamochi. Drawing clustered graphs on an orthogonal grid. *Journal of Graph Algorithms and Applications*, 3(4):3–29, 1999.
- [EKLN04] Cesim Erten, Stephen G. Kobourov, Vu Le, and Armand Navabi. Simultaneous graph drawing: Layout algorithms and visualization schemes. In *Proceedings of the 11th International Symposium on Graph Drawing (GD'03)*, volume 2912 of *Lecture Notes in Computer Science*, pages 437–449. Springer-Verlag, 2004.
- [FE02] Carsten Friedrich and Peter Eades. Graph drawing in motion. *Journal of Graph Algorithms and Applications*, 6(3):353–370, 2002.
- [FK46] Elaine Forsyth and Leo Katz. A matrix approach to the analysis of sociometric

- data: Preliminary report. *Sociometry*, 9:340–347, 1946.
- [Fre79] Linton C. Freeman. Centrality in social networks: Conceptual clarification I. *Social Networks*, 1:215–239, 1979.
  - [Fre00] Linton C. Freeman. Visualizing social networks. *Journal of Social Structure*, 1(1), 2000.
  - [Fre04a] Linton C. Freeman. *The Development of Social Network Analysis: A Study in the Sociology of Science*. Empirical Press, 2004.
  - [Fre04b] Ralph Freese. Automated lattice drawing. In *Proceedings of the 2nd International Conference on Formal Concept Analysis (ICFCA'04)*, volume 2961 of *Lecture Notes in Computer Science*, pages 112–127. Springer-Verlag, 2004.
  - [Fre05] Linton C. Freeman. Graphic techniques for exploring social network data. In Carrington et al. [CSW05], pages 248–269.
  - [Fre08] Linton C. Freeman, editor. *Social Network Analysis*. Sage, 2008. Volumes I–IV.
  - [Fri91] Noah E. Friedkin. Theoretical foundations for centrality measures. *American Journal of Sociology*, 96(6):1478–1504, May 1991.
  - [FT08] Yaniv Frishman and Ayellet Tal. Online dynamic graph drawing. *IEEE Transactions on Visualization and Computer Graphics*, 14(4):727–740, 2008.
  - [Fur10] Borko Furht. *Handbook of Social Network Technologies and Applications*. Springer-Verlag, 2010.
  - [FW93] Linton C. Freeman and Douglas R. White. Using Galois lattices to represent network data. *Sociological Methodology*, 23:127–146, 1993.
  - [GFC05] Mohammad Ghoniem, Jean-Daniel Fekete, and Philippe Castagliola. On the readability of graphs using node-link and matrix-based representations: a controlled experiment and statistical analysis. *Information Visualization*, 4(2):114–135, 2005.
  - [GKN04] Emden R. Gansner, Yehuda Koren, and Stephen C. North. Graph drawing by stress majorization. In *Proceedings of the 12th International Symposium on Graph Drawing (GD'04)*, volume 3383 of *Lecture Notes in Computer Science*, pages 239–250. Springer-Verlag, 2004.
  - [GW98] Bernhard Ganter and Rudolf Wille. *Formal Concept Analysis: Mathematical Foundations*. Springer-Verlag, 1998.
  - [HB05] Jeffrey Heer and Danah Boyd. Vizster: Visualizing online social networks. In *Proceedings of IEEE Symposium of Information Visualization (InfoVis'05)*, pages 32–39, 2005.
  - [HEH09] Weidong Huang, Peter Eades, and Seokhee Hong. Measuring effectiveness of graph visualizations: A cognitive load approach. *Information Visualization*, 8(3):139–152, 2009.
  - [HEW98] Mao Lin Huang, Peter Eades, and Junhu Wang. On-line animated visualization of huge graphs using a modified spring algorithm. *Journal of Visual Languages and Computing*, 9(6):623–645, 1998.
  - [HF06] Nathalie Henry and Jean-Daniel Fekete. MatrixExplorer: a dual-representation system to explore social networks. *IEEE Transactions on Visualization and Computer Graphics*, 12(5):677–684, 2006.
  - [HF07] Nathalie Henry and Jean-Daniel Fekete. MatLink: Enhanced matrix visualization for analyzing social networks. In *Proceedings of the 11th IFIP Conference on Human-Computer Interaction (INTERACT 2007)*, volume 4663 of *Lecture Notes in Computer Science*, pages 288–302. Springer-Verlag, 2007.

- [HFM07] Nathalie Henry, Jean-Daniel Fekete, and Michael J. McGuffin. NodeTriX: a hybrid visualization of social networks. *IEEE Transactions on Visualization and Computer Graphics*, 13(6):1302–1309, 2007.
- [HHE07] Weidong Huang, Seokhee Hong, and Peter Eades. Effects of sociogram drawing conventions and edge crossings in social network visualization. *Journal of Graph Algorithms and Applications*, 11(2):397–429, 2007.
- [HN07] Mao Lin Huang and Quang Vinh Nguyen. A space efficient clustered visualization of large graphs. In *Proceedings of the 4th International Conference on Image and Graphics (ICIG '07)*, pages 920–927, 2007.
- [Hob03] Robert Hobbs. *Mark Lombard: Global Networks*. Independent Curators International, New York, 2003.
- [Hol06] Danny Holten. Hierarchical edge bundles: Visualization of adjacency relations in hierarchical data. *IEEE Transactions on Visualization and Computer Graphics*, 12(5):741–748, 2006.
- [HSS10] Derek Hansen, Ben Shneiderman, and Marc A. Smith. *Analyzing Social Media Networks with NodeXL: Insights from a Connected World*. Morgan Kaufmann, 2010.
- [HvD11] Mark Huisman and Marijtje A. J. van Duijn. A reader’s guide to SNA software. In John Scott and Peter J. Carrington, editors, *The SAGE Handbook of Social Network Analysis*, pages 578–600. Sage, 2011.
- [IMMS09] Takayuki Itoh, Chris Muelder, Kwan-Liu Ma, and Jun Sese. A hybrid space-filling and force-directed layout method for visualizing multiple-category graphs. In *Proceedings of the IEEE Pacific Visualization Symposium (PacificVis'09)*, pages 121–128, 2009.
- [JHGH08] Yuntao Jia, Jared Hoberock, Michael Garland, and John C. Hart. On the visualization of social and other scale-free networks. *IEEE Transactions on Visualization and Computer Graphics*, 14(6):1285–1292, 2008.
- [JK04] Jeffrey C. Johnson and Lothar Krempel. Network visualization: The “Bush Team” in Reuters news ticker 9/11–11/15/01. *Journal of Social Structure*, 5(1), 2004.
- [JM04] Michael Jünger and Petra Mutzel, editors. *Graph Drawing Software*. Springer-Verlag, 2004.
- [JP87] David S. Johnson and Henry O. Pollak. Hypergraph planarity and the complexity of drawing Venn diagrams. *Journal of Graph Theory*, 11(3):309–325, 1987.
- [Kam89] Tomihisa Kamada. *Visualizing Abstract Objects and Relations*. World Scientific, 1989.
- [Kat47] Leo Katz. On the matrix analysis of sociometric data. *Sociometry*, 10:233–241, 1947.
- [Kat53] Leo Katz. A new status index derived from sociometric analysis. *Psychometrika*, 18(1):39–43, 1953.
- [KB83] David Knoke and Ronald S. Burt. Prominence. In Ronald S. Burt and Michael J. Minor, editors, *Applied Network Analysis*, pages 195–222. Sage Publications, 1983.
- [KH05] Yehuda Koren and David Harel. One-dimensional layout optimization, with applications to graph drawing by axis separation. *Computational Geometry*, 32(2):115–138, 2005.
- [Klo81] Alden S. Klov Dahl. A note on images of networks. *Social Networks*, 3:197–214, 1981.
- [Kra96] David Krackhardt. Social networks and the liability of newness for managers.

- In C. L. Cooper and D. M. Rousseau, editors, *Trends in Organizational Behavior*, volume 3, pages 159–173. John Wiley & Sons, 1996.
- [KS04] Dirk Koschützki and Falk Schreiber. Comparison of centralities for biological networks. In *Proceedings of the German Conference on Bioinformatics 2004*, volume P-53 of *Lecture Notes in Informatics*, pages 199–206. Springer-Verlag, 2004.
- [KvKS09] Michael Kaufmann, Marc van Kreveld, and Bettina Speckmann. Subdivision drawings of hypergraphs. In *Proceedings of the 16th International Symposium on Graph Drawing (GD’08)*, volume 5417 of *Lecture Notes in Computer Science*, pages 396–407. Springer-Verlag, 2009.
- [KZ09] Pushpa Kumar and Kang Zhang. Node overlap removal in clustered directed acyclic graphs. *Journal of Visual Languages and Computing*, 20(6):403–419, 2009.
- [Ler05] Jürgen Lerner. Role assignments. In Brandes and Erlebach [BE05], pages 216–252.
- [LG66] Edward O. Laumann and Louis Guttman. The relative associational contiguity of occupations in an urban setting. *American Sociological Review*, 31:169–178, 1966.
- [LMR98] Kelly A. Lyons, Henk Meijer, and David Rappaport. Algorithms for cluster busting in anchored graph drawing. *Journal of Graph Algorithms and Applications*, 2(1):1–24, 1998.
- [Lon48] T. Wilson Longmore. A matrix approach to the analysis of rank and status in a community in peru. *Sociometry*, 11(3):192–206, 1948.
- [LP49] R. Duncan Luce and Albert Perry. A method of matrix analysis of group structure. *Psychometrika*, 14:95–116, 1949.
- [LPA<sup>+</sup>09] David Lazer, Alex Pentland, Lada Adamic, Sinan Aral, Albert-László Barabási, Devon Brewer, Nicholas Christakis, and Noshir Contractor, James Fowler, Myron Gutmann, Tony Jebara, Gary King, Michael Macy, Deb Roy, and Marshall Van Alstyne. Computational social science. *Science*, 323(5915):721–723, 2009.
- [Mäk90] Erkki Mäkinen. How to draw a hypergraph. *International Journal of Computer Mathematics*, 34(3-4):177–185, 1990.
- [MBK97] Cathleen McGrath, Jim Blythe, and David Krackhardt. The effect of spatial arrangement on judgments and errors in interpreting graphs. *Social Networks*, 19(3):223–242, 1997.
- [MELS95] Kazuo Misue, Peter Eades, Wei Lai, and Kozo Sugiyama. Layout adjustment and the mental map. *Journal on Visual Languages and Computing*, 6(2):183–210, 1995.
- [MH01] Matthias Müller-Hannemann. Drawing trees, series-parallel digraphs, and lattices. In Michael Kaufmann and Dorothea Wagner, editors, *Drawing Graphs: Methods and Models*, volume 2025 of *Lecture Notes in Computer Science*, pages 46–70. Springer, 2001.
- [Mis07] Kazuo Misue. Anchored maps: Visualization techniques for drawing bipartite graphs. In *The Human-Computer Interaction International Conference Proceedings (HCII 2007)*, volume 4551 of *Lecture Notes in Computer Science*, pages 106–114. Springer-Verlag, 2007.
- [Mit94] J. Clyde Mitchell. Situational analysis and network analysis. *Connections*, 17(1):16–22, 1994.
- [MMBd05] James Moody, Daniel A. McFarland, and Skye Bender-deMoll. Dynamic network visualization. *American Journal of Sociology*, 110(4):1206–1241, 2005.

- [Mor46] Jakob L. Moreno. Sociogram and sociomatrix: a note to the paper by Forsyth and Katz. *Sociometry*, 9:348–349, 1946.
- [Mor53] Jakob L. Moreno. *Who Shall Survive? Foundations of Sociometry, Group Psychotherapy, and Sociodrama*. Beacon House, 1953. Originally published in 1934.
- [New61] Theodore M. Newcomb. *The Acquaintance Process*. Holt, Rinehart & Winston, 1961.
- [Nor40] Mary L. Northway. A method for depicting social relationships obtained by sociometric testing. *Sociometry*, 3(2):144–150, 1940.
- [Nor52] Mary L. Northway. *A Primer of Sociometry*. University of Toronto Press, 1952.
- [Nor54] Mary L. Northway. A plan for sociometric studies in a longitudinal programme of research in child development. *Sociometry*, 17(3):272–281, 1954.
- [Nor96] Stephen C. North. Incremental layout with DynaDag. In Brandenburg [Bra96], pages 409–418.
- [OS07] Hiroki Omote and Kozo Sugiyama. Method for visualizing complicated structures based on unified simplification strategies. *IEICE Transactions on Information and Systems*, E90-D(10):1649–1656, 2007.
- [PCJ97] Helen C. Purchase, Robert F. Cohen, and Murray James. An experimental study of the basis for graph drawing algorithms. *ACM Journal of Experimental Algorithmics*, 2(4), 1997.
- [Pet99] W. M. Flinders Petrie. Sequences in prehistoric remains. *Journal of the Anthropological Institute of Great Britain and Ireland*, 29(3/4):295–301, 1899.
- [PNK10] Ruth Pfosser, Helmut Neundlinger, and Harald Katzmaier. Die unverbrüchliche Solidarität in einer Extremsituation und die Fähigkeit, sich zu befreien. Das ist Spanien. *Der Standard*, 13. Juli, 2010.
- [PNR08] Christian Pich, Lev Nachmanson, and George G. Robertson. Visual analysis of importance and grouping in software dependency graphs. In *Proceedings of the ACM 2008 Symposium on Software Visualization (SOFTVIS 2008)*, pages 29–32, 2008.
- [PS08] Helen C. Purchase and Amanjit Samra. Extremes are better: Investigating mental map preservation in dynamic graphs. In *Proceedings of the 5th International Conference on Diagrammatic Representation and Inference (Diagrams 2008)*, volume 5223 of *Lecture Notes in Computer Science*, pages 60–73. Springer-Verlag, 2008.
- [RCE06] Jon Ducrou Richard Cole and Peter Eklund. Automated layout of small lattices using layer diagrams. In *Proceedings of the 4th International Conference on Formal Concept Analysis (ICFCA '06)*, volume 3874 of *Lecture Notes in Computer Science*, pages 291–305. Springer-Verlag, 2006.
- [RF10] Nathalie Riche and Jean-Daniel Fekete. Novel visualizations and interactions for social networks exploration. In *Handbook of Social Network Technologies and Applications* [Fur10], pages 611–636.
- [RZF08] Peter Rodgers, Leishi Zhang, and Andrew Fish. General Euler diagram generation. In *Proceedings of the 5th International Conference on the Theory and Application of Diagrams (Diagrams'08)*, volume 5223 of *Lecture Notes in Artificial Intelligence*, pages 13–27. Springer-Verlag, 2008.
- [SA06] Ben Shneiderman and Aleks Aris. Network visualization by semantic sub-graphs. *IEEE Transactions on Visualization and Computer Graphics*, 12(5):733–740, 2006.



- [SAA09] Paolo Simonetto, David Auber, and Daniel Archambault. Fully automatic visualisation of overlapping sets. *Computer Graphics Forum*, 28(3):967–974, 2009.
- [Sco00] John Scott. *Social Network Analysis: A Handbook*. Sage, 2nd edition, 2000.
- [Sei83] Stephen B. Seidman. Network structure and minimum degree. *Social Networks*, 5:269–287, 1983.
- [SJUS08] Hans-Jörg Schulz, Mathias John, Andrea Unger, and Heidrun Schumann. Visual analysis of bipartite biological networks. In *Proceedings of the Eurographics Workshop on Visual Computing for Biomedicine (VCBM 2008)*, pages 135–142. Eurographics, 2008.
- [SM07] Zeqian Shen and Kwan-Liu Ma. Path visualization for adjacency matrices. In *Proceedings of the 9th Eurographics/IEEE-VGTC Symposium on Visualization (EuroVis '07)*, pages 83–90. Eurographics, 2007.
- [ST10] Rodrigo Santamaría and Roberto Therón. Visualization of intersecting groups based on hypergraphs. *IEICE Transactions on Information and Systems*, E93-D(7):1957–1964, 2010.
- [Tuf83] Edward R. Tufte. *The Visual Display of Quantitative Information*. Graphics Press, 1983.
- [vHSD09] Frank van Ham, Hans-Jörg Schulz, and Joan M. Dimicco. Honeycomb: Visual analysis of large scale social networks. In *Proceedings of the 13th IFIP Conference on Human-Computer Interaction (INTERACT 2009)*, volume 5727 of *Lecture Notes in Computer Science*, pages 429–442. Springer-Verlag, 2009.
- [vHW08] Frank van Ham and Martin Wattenberg. Centrality based visualization of small world graphs. *Computer Graphics Forum*, 27(3):975–982, 2008.
- [Wat06] Martin Wattenberg. Visual exploration of multivariate graphs. In *Proceedings of the SIGCHI Conference on Human Factors in Computing Systems (CHI '06)*, pages 811–819, 2006.
- [WB88] Barry Wellman and Stephen D. Berkowitz, editors. *Social Structures: A Network Approach*. Cambridge University Press, 1988.
- [Wer44] Max Wertheimer. *Gestalt Theory*. Hayer Barton Press, 1944.
- [WF94] Stanley Wasserman and Katherine Faust. *Social Network Analysis: Methods and Applications*. Cambridge University Press, 1994.
- [WG98] Karl Erich Wolff and Siegfried Gabler. Comparison of visualizations in formal concept analysis and correspondence analysis. In Michael Greenacre and Jörg Blasius, editors, *Visualization of Categorical Data*, pages 85–97. Academic Press, 1998.
- [Why43] William F. Whyte. *Street Corner Society*. University of Chicago Press, 1943.
- [Win94] William D. Winn. Contributions of perceptual and cognitive processes to the comprehension of graphics. In W. Schnotz and R.W. Kulhavy, editors, *Comprehension of Graphics*, number 108 in *Advances in Psychology*, pages 3–27. Elsevier, 1994.
- [WM96] Xiaobo Wang and Isao Miyamoto. Generating customized layouts. In Brandenburg [Bra96], pages 504–515.
- [Wol07] Alexander Wolff. Drawing subway maps: Asurvey. *Informatik - Forschung und Entwicklung*, 22(1):23–44, 2007.
- [XTT<sup>+</sup>10] Kaikuo Xu, Changjie Tang, Rong Tang, Ghulam Ali, and Jun Zhu. A comparative study of six software packages for complex network research. In *Proceedings of the 2nd International Conference on Communication Software and Networks (ICCSN '10)*, pages 350–354, 2010.

- [ZSE05] Lanbo Zheng, Le Song, and Peter Eades. Crossing minimization problems of drawing bipartite graphs in two clusters. In *Proceedings of the 2005 Asia-Pacific Symposium on Information Visualisation (APVis '05)*, pages 33–37. Australian Computer Society, 2005.